Escape-limited maximum energy at perpendicular shocks in the interstellar magnetic field

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Cosmic Rays and Cosmic Ray Escape

cosmic rays(CRs) up to $10^{15.5}$ eV

We have not understood

What types of SNRs can accelerate CRs to PeV?

Which shock accelerates Galactic CRs,

parallel shocks or **perpendicular shocks**?

cosmic ray escape

The CR escape process is important to determine the maximum energy and the spectral index of the CR energy spectrum.

(Ptuskin & Zirakashvili 2003, 2005, Ohira, Murase, Yamazaki 2010, Ohira & Ioka 2011)

Previous studies assume **the diffusion approximation.**

The gyration is crucial to realize the rapid acceleration in the perp. shock.

(Takamoto & Kirk 2015, Kamijima et al.2020)

We investigate the escape process and the escape-limited maximum energy at the perp. shock without the diffusion approximation.

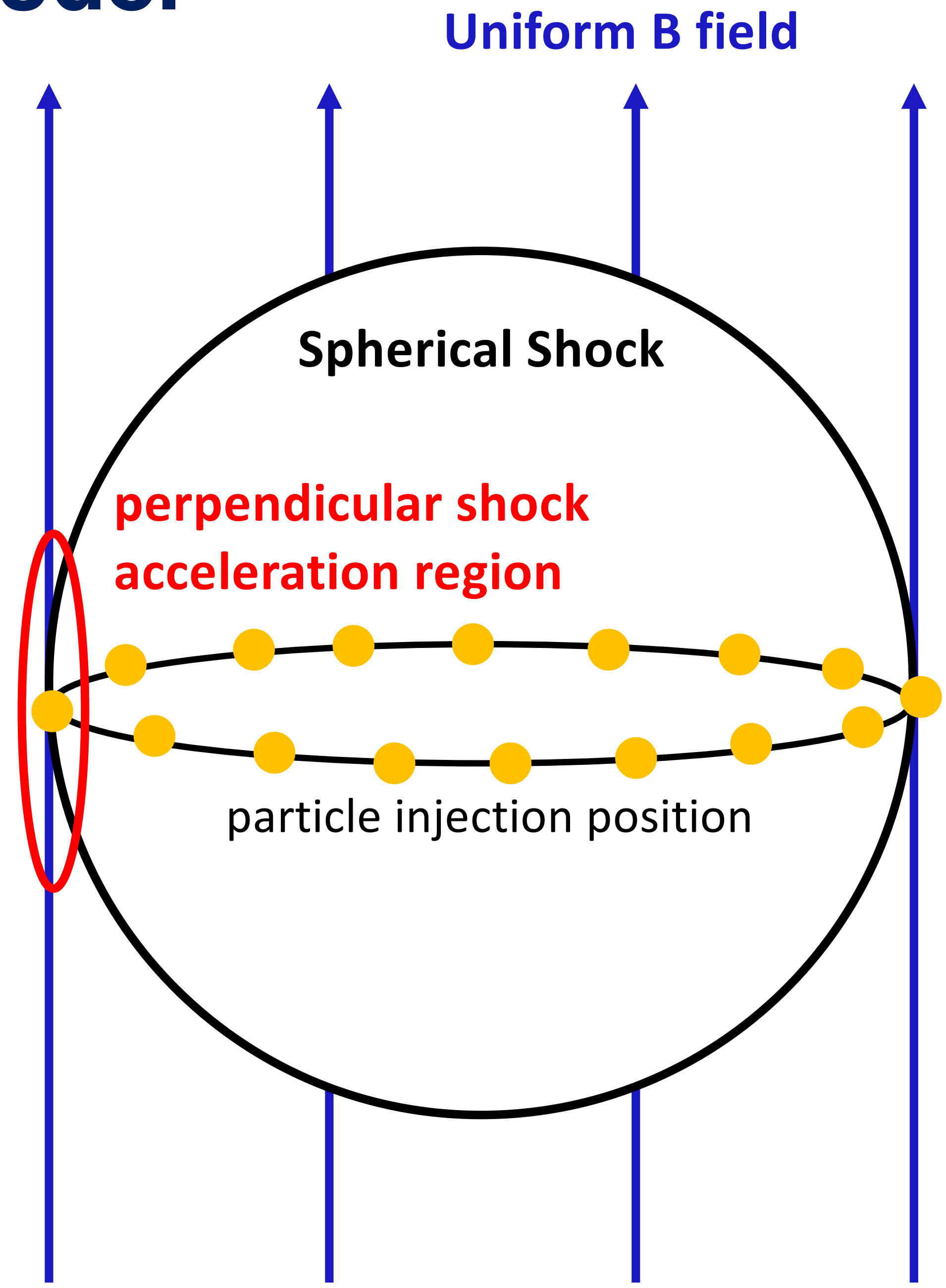
Our Model

System：
uniform B field＋Spherical shock
(assumption: turbulent B field in the downstream region)

Escape-limited maximum energy : $E_{\mathrm{max,esc}}$
(acceleration time, t_{acc}, = escape time ,t_{esc})

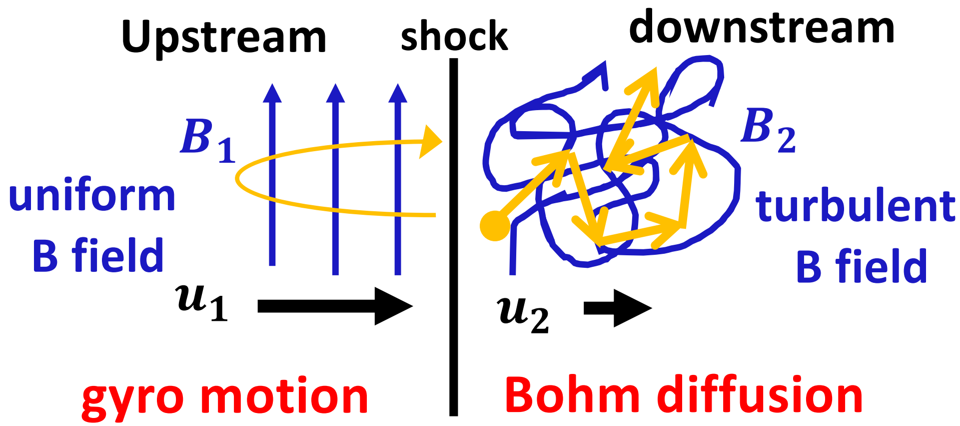
- Perpendicular shock acceleration model (Kamijima, Ohira, Yamazaki 2020)
  → acceleration time t_{acc}

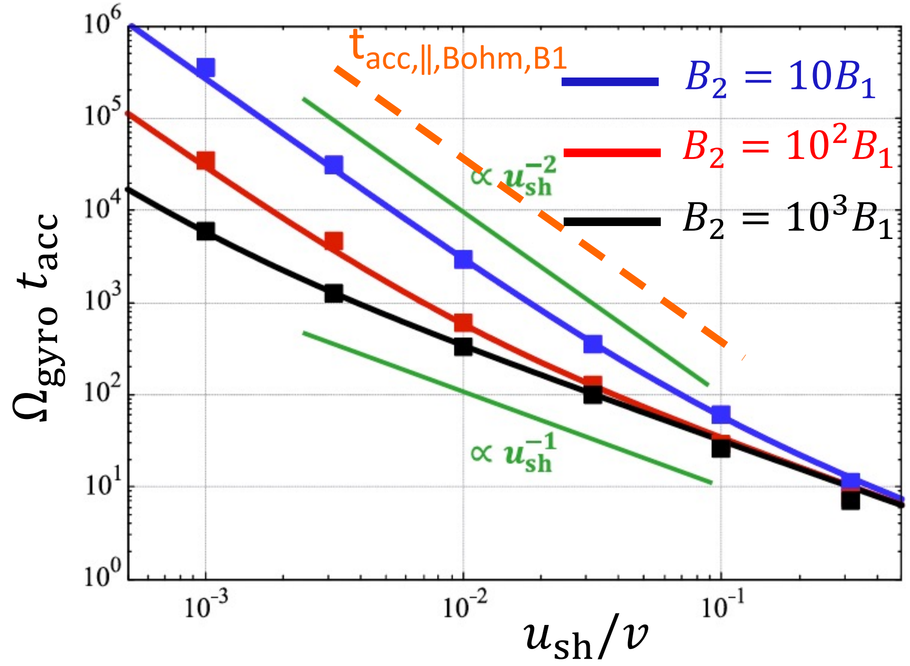
- the size of acceleration region: R_{acc}
- particle motion in the acceleration region
  → escape time t_{esc}



Perpendicular Shock Acceleration Model

(Kamijima, Ohira, Yamazaki, 2020)





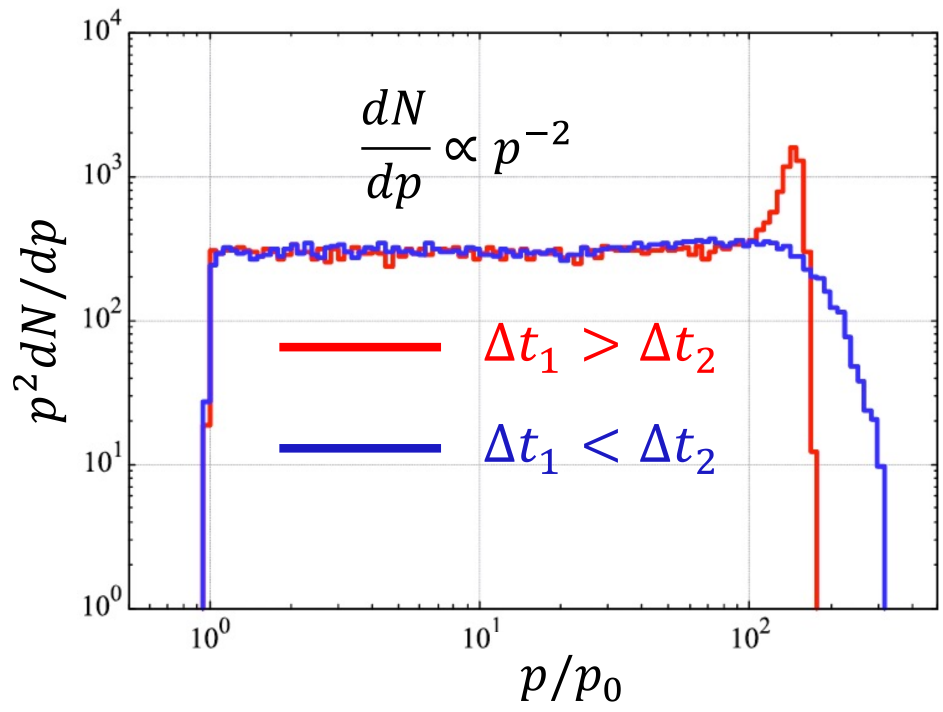
residence time

$$\Delta t_1 = \pi \Omega_{\mathrm{g},1}^{-1} \qquad \Delta t_2 = \frac{4\kappa_2}{u_2 v} \qquad \text{Drury 1983}$$

aceleration time

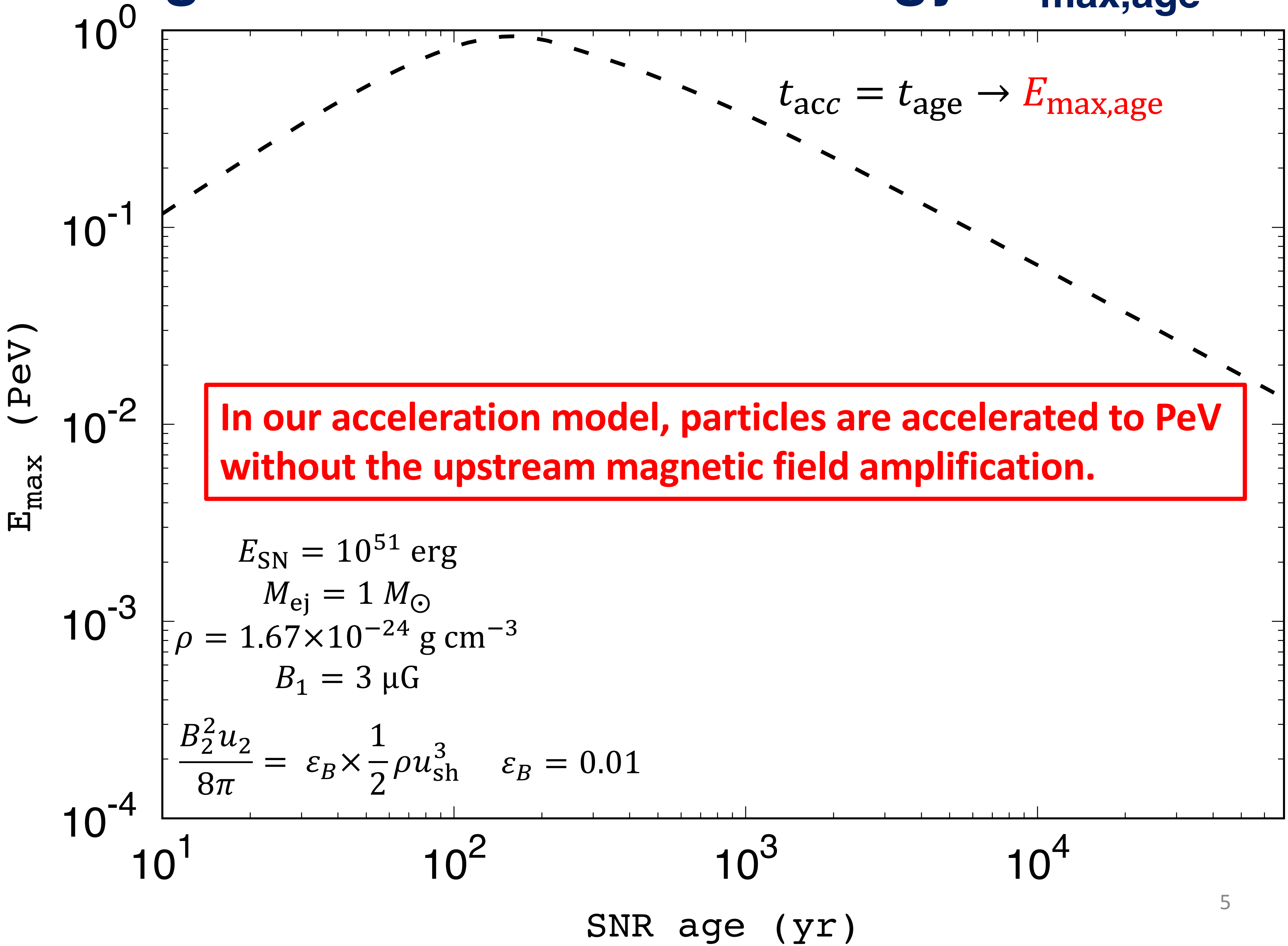
$$t_{\mathrm{acc}} = (\Delta t_1 + \Delta t_2) p/\Delta p \qquad \frac{\Delta p}{p} = \frac{u_1}{v}$$

$$t_{\mathrm{acc}} = \pi \left(\frac{u_{\mathrm{sh}}}{v}\right)^{-1} \Omega_{\mathrm{g},1}^{-1} + \frac{16}{3}\left(\frac{B_2}{B_1}\right)^{-1} \left(\frac{u_{\mathrm{sh}}}{v}\right)^{-2} \Omega_{\mathrm{g},1}^{-1}$$



Our model can realize the rapid acceleration and the canonical spectrum, $dN/dp \propto p^{-2}$, simultaneously.

Age-limited Maximum Energy: $E_{max,age}$



Particle Motion in Acceleration Region

Particles are **not scattered in the upstream region,** and are **scattered in the downstream region.**

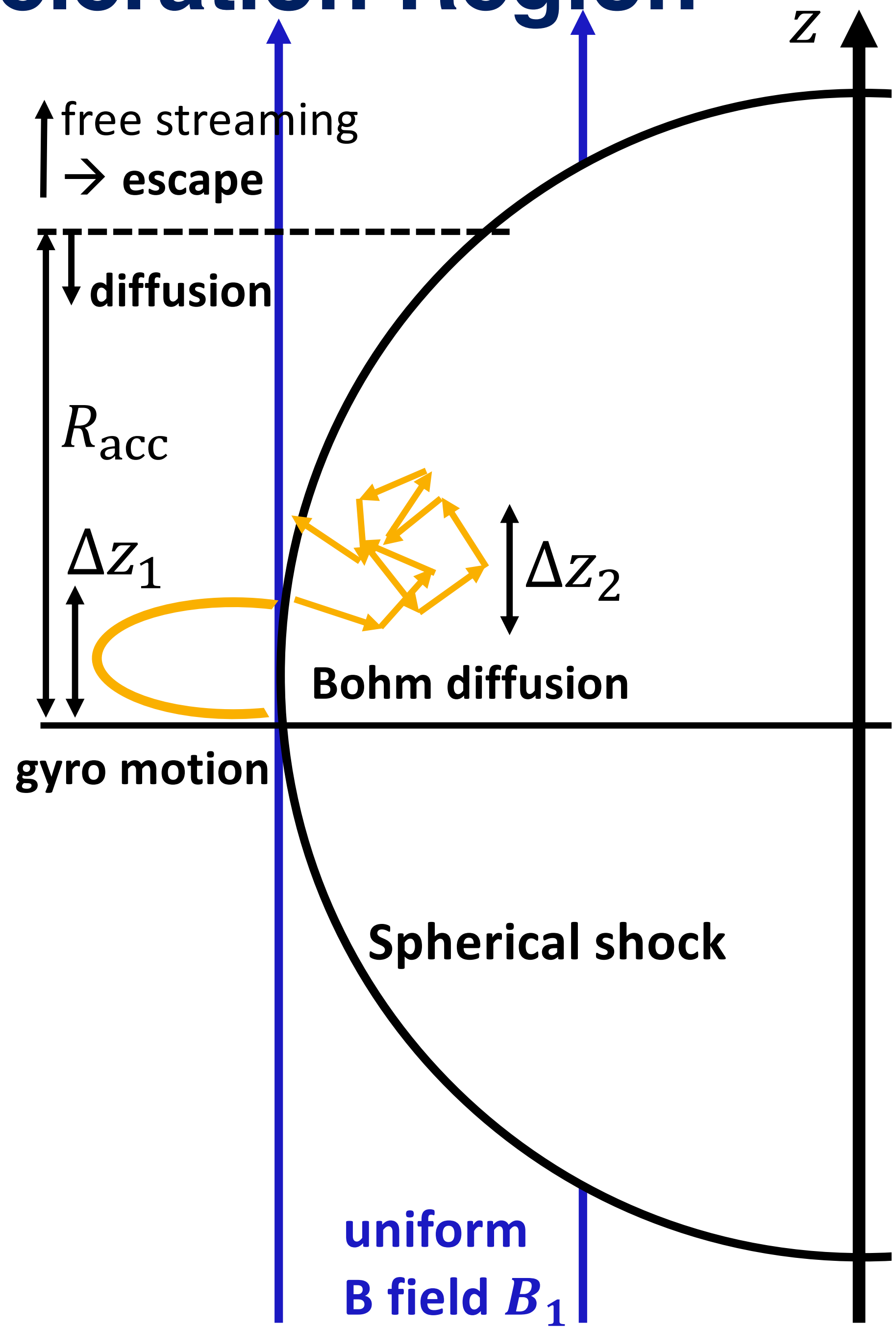
The particle motion for longer time scale than the one cycle time is regarded as **the diffusion.**

From the relation between the diffusion length and time , $z^2 \sim 2\kappa_{zz} t$, the escape time is

$$t_{\rm esc} = \frac{R_{\rm acc}^2}{2\kappa_{\rm zz}}$$

size of the acceleration region: $R_{\rm acc}$

diffusion coefficient along B_1: $\kappa_{\rm zz}$



Particle Motion in Acceleration Region

diffusion coefficient along B_1: κ_{zz}

$$\kappa_{zz} = \frac{(\Delta z_1)^2 + (\Delta z_2)^2}{2(\Delta t_1 + \Delta t_2)}$$

mean residence time : Δt_1, Δt_2

$$\Delta t_1 = \pi \Omega_{\mathrm{g},1}^{-1}$$

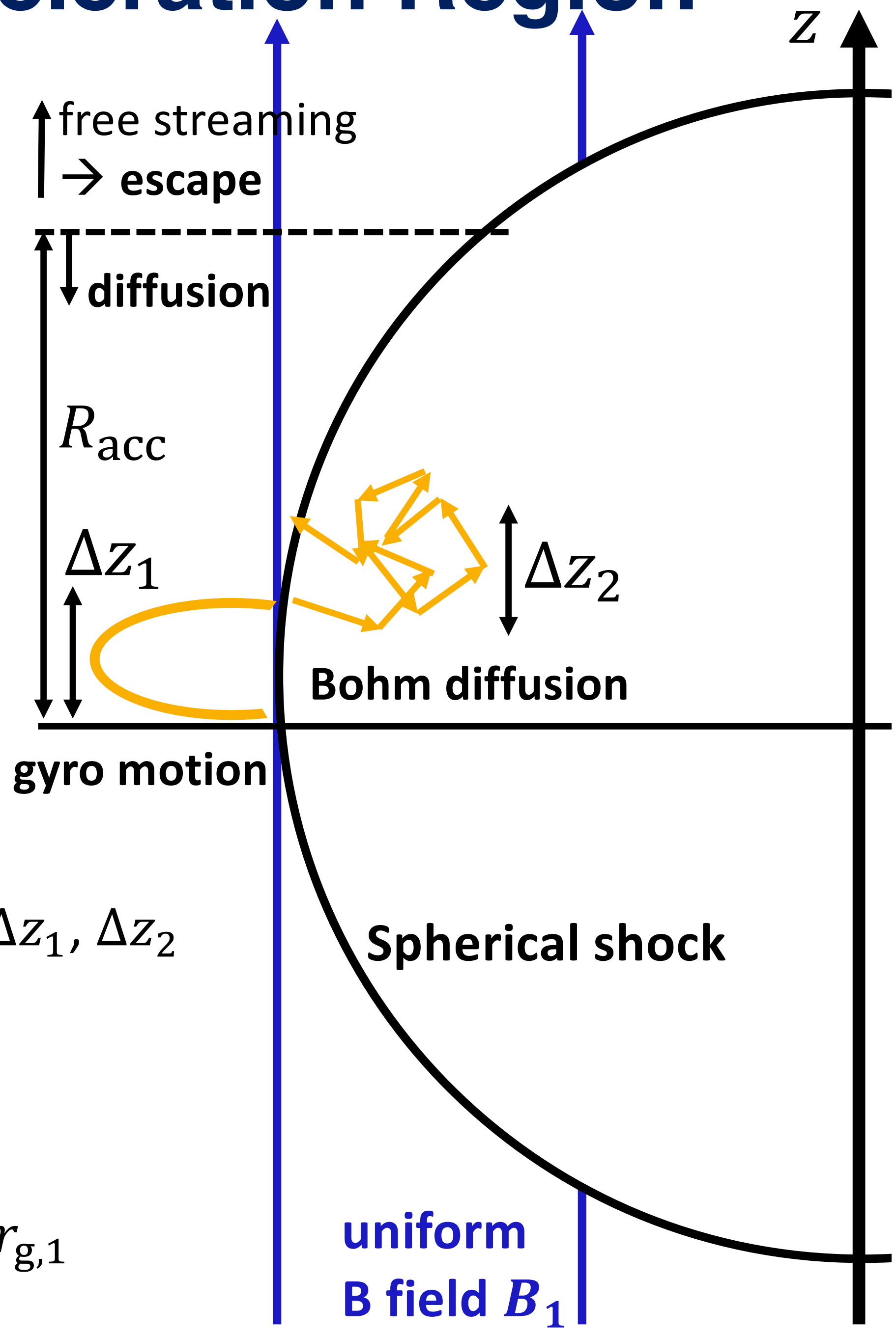
$$\Delta t_2 = \frac{4\kappa_2}{u_2 v} = \frac{4r}{3}\left(\frac{u_{\mathrm{sh}}}{v}\right)^{-1}\left(\frac{B_2}{B_1}\right)^{-1}\Omega_{\mathrm{g},1}^{-1}$$

$$\kappa_2 = \frac{1}{3} r_{\mathrm{g},2} v = \frac{1}{3}\left(\frac{B_2}{B_1}\right)^{-1} r_{\mathrm{g},1} v$$

mean displacement during the residence time: Δz_1, Δz_2

$$\Delta z_1 = \sqrt{\langle v_z^2 \rangle}\,\Delta t_1 = \frac{v}{2}\Delta t_1 = \frac{\pi}{2} r_{\mathrm{g},1}$$

$$\Delta z_2 = \sqrt{2\kappa_2 \Delta t_2} = \frac{\sqrt{8r}}{3}\left(\frac{u_{\mathrm{sh}}}{v}\right)^{-1/2}\left(\frac{B_2}{B_1}\right)^{-1} r_{\mathrm{g},1}$$



$(r_{\mathrm{g},1} = v\Omega_{\mathrm{g},1}^{-1})$

Size of Acceleration Region

Here, we consider accelerating particles.

In the **subluminal shock**, particles can **escape to the far upstream region**.

We consider the escape probability that particles escape to the far upstream region during the one cycle time of DSA , $P_{\rm esc}$.

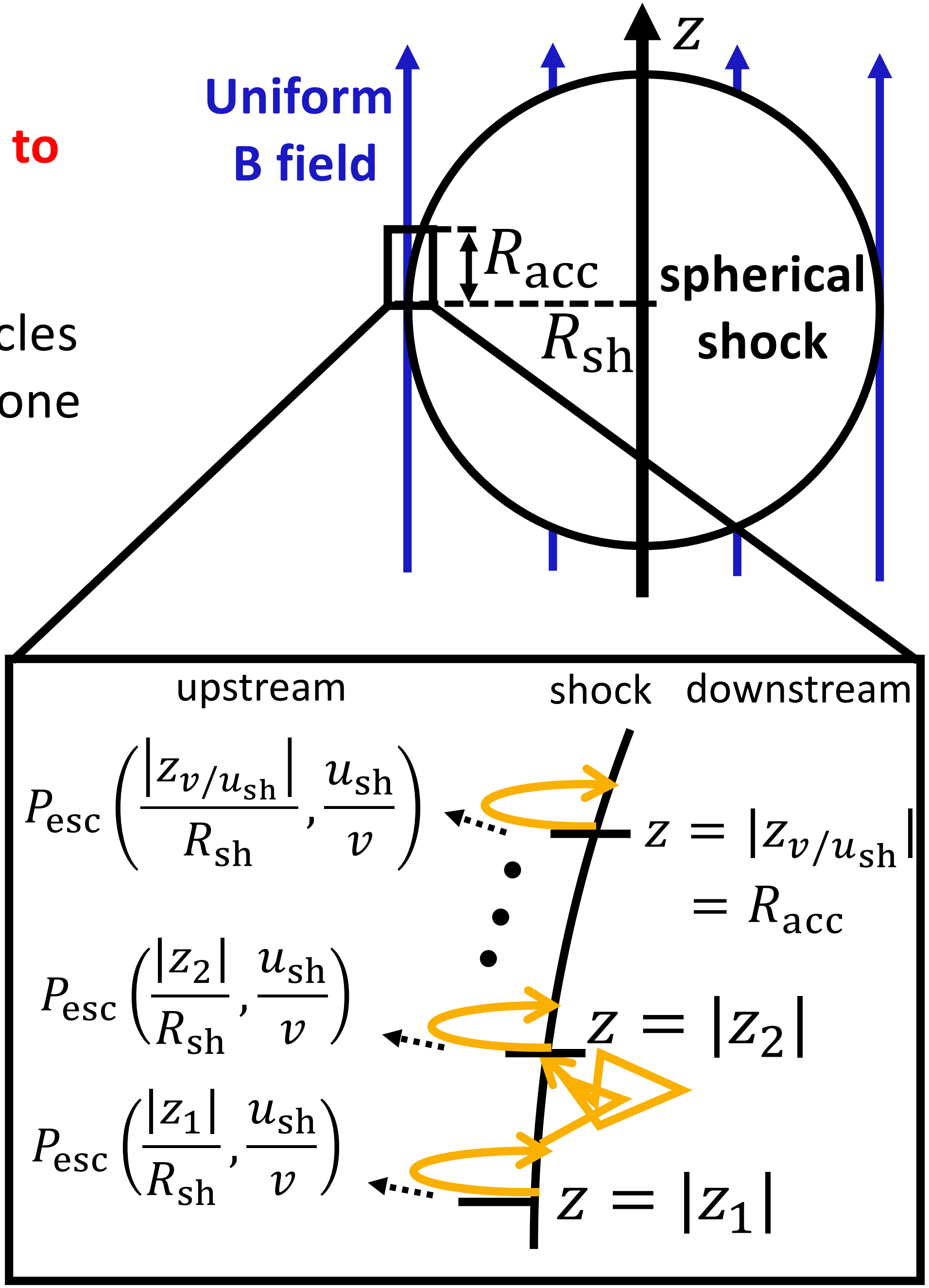
The escape probability at the point of the i-th shock crossing, $|z_i|$, is

$$P_{\rm esc}\left(\frac{|z_i|}{R_{\rm sh}}, \frac{u_{\rm sh}}{v}\right)$$

The size of the acceleration region is given by

$$P_{\rm acc} = \prod_{i=1}^{v/u_{\rm sh}} \left[1 - P_{\rm esc}\left(\frac{|z_i|}{R_{\rm sh}}, \frac{u_{\rm sh}}{v}\right)\right] \approx 0$$

The return probability at $z = |z_i|$



Size of Acceleration Region

$$P_{\mathrm{acc}} = \prod_{i=1}^{v/u_{\mathrm{sh}}} \left[1 - P_{\mathrm{esc}}\left(\frac{|z_i|}{R_{\mathrm{sh}}}, \frac{u_{\mathrm{sh}}}{v}\right)\right] \approx 0$$

We need the complex calculation to get $\left|z_{v/u_{\mathrm{sh}}}\right| = R_{\mathrm{acc}}$.
To simplify, we assume that the particles are $z = R_{\mathrm{acc}}$ during the acceleration.

$$P_{\mathrm{acc}} \approx \left[1 - P_{\mathrm{esc}}\left(\frac{R_{\mathrm{acc}}}{R_{\mathrm{sh}}}, \frac{u_{\mathrm{sh}}}{v}\right)\right]^{\frac{v}{u_{\mathrm{sh}}}} \approx 0$$

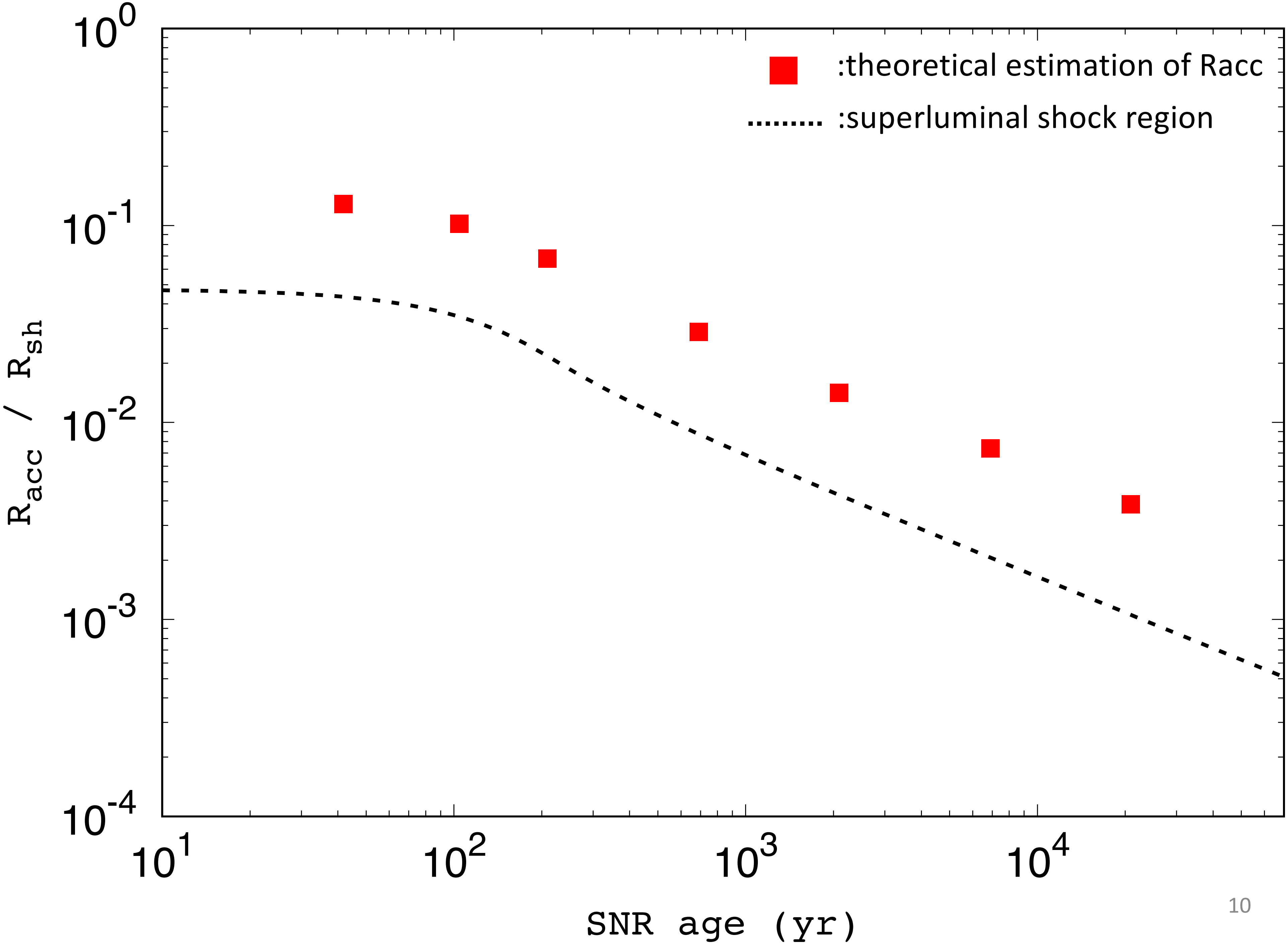
$$P_{\mathrm{esc}}\left(\frac{R_{\mathrm{acc}}}{R_{\mathrm{sh}}}, \frac{u_{\mathrm{sh}}}{v}\right) \ll 1$$

$$\leftrightarrow \; 1 - \frac{v}{u_{\mathrm{sh}}} P_{\mathrm{esc}}\left(\frac{R_{\mathrm{acc}}}{R_{\mathrm{sh}}}, \frac{u_{\mathrm{sh}}}{v}\right) \approx 0$$

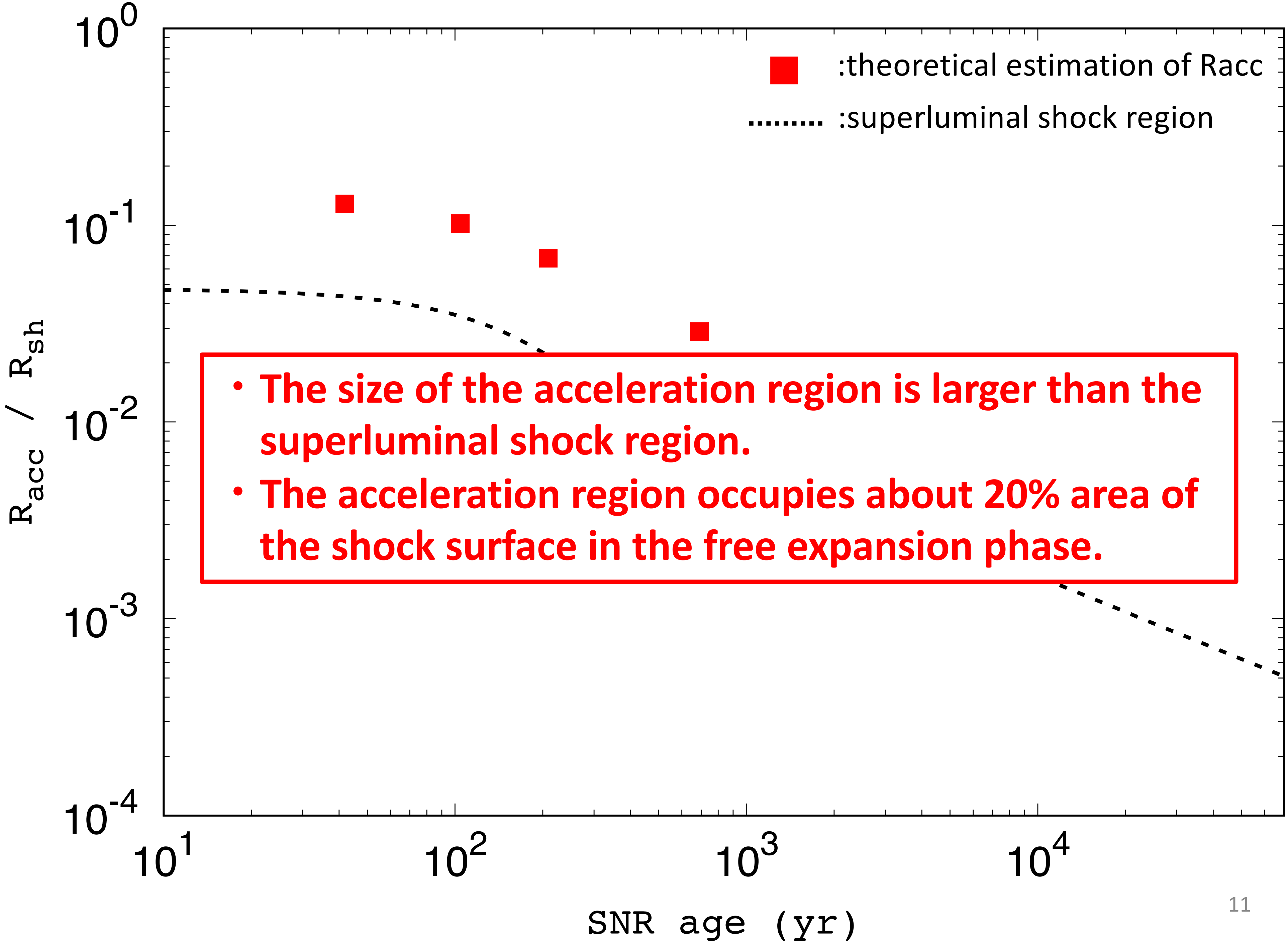
$$\boxed{P_{\mathrm{esc}}\left(\frac{R_{\mathrm{acc}}}{R_{\mathrm{sh}}}, \frac{u_{\mathrm{sh}}}{v}\right) = \frac{u_{\mathrm{sh}}}{v}}$$

We numerically investigate P_{esc} for several $|z|/R_{\mathrm{sh}}$ and u_{sh}/v, and estimate $|z| = R_{\mathrm{acc}}$ given by the condition $P_{\mathrm{esc}}\left(\frac{|z|}{R_{\mathrm{sh}}}, \frac{u_{\mathrm{sh}}}{v}\right) = \frac{u_{\mathrm{sh}}}{v}$.

Size of Acceleration Region

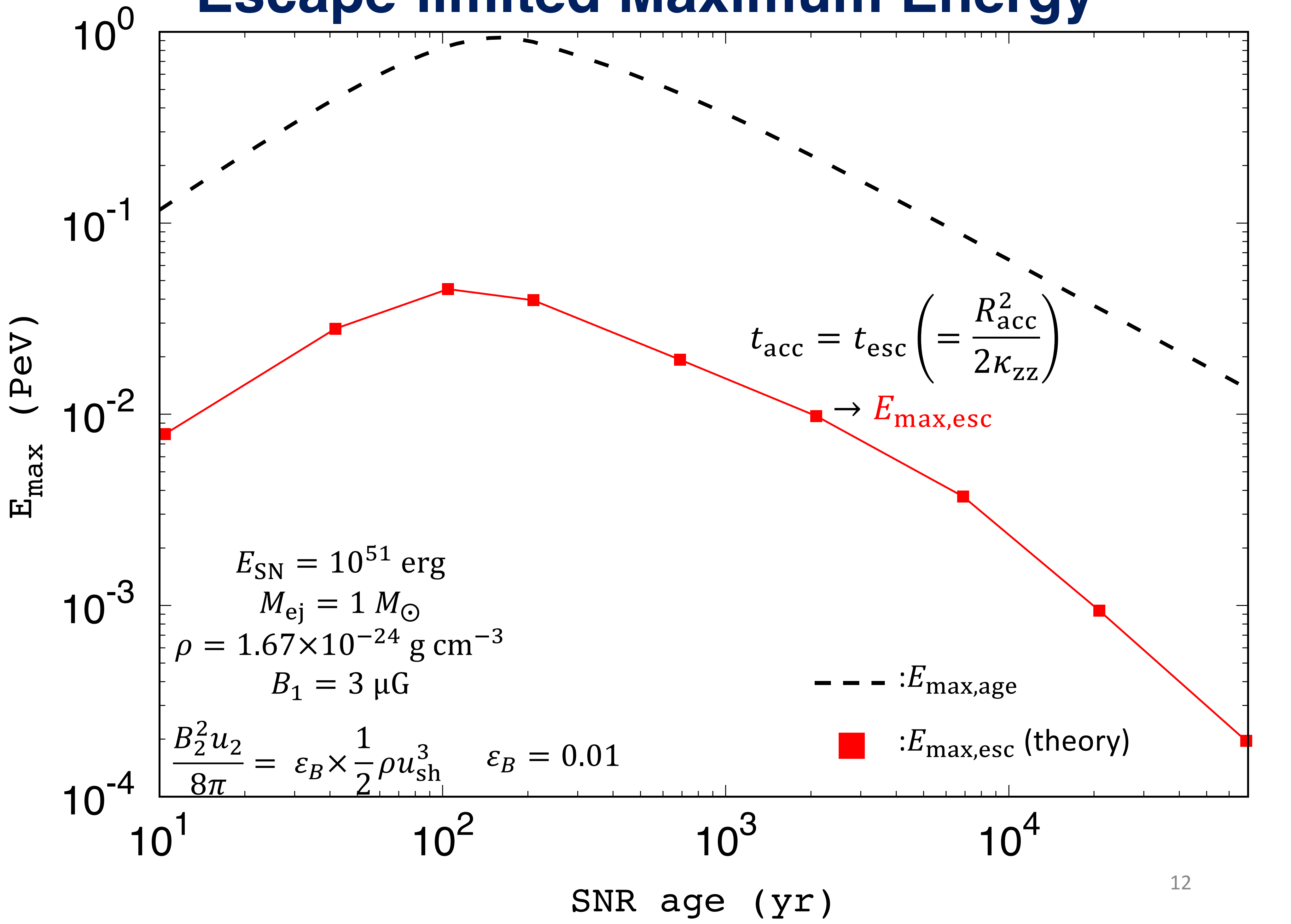


Size of Acceleration Region

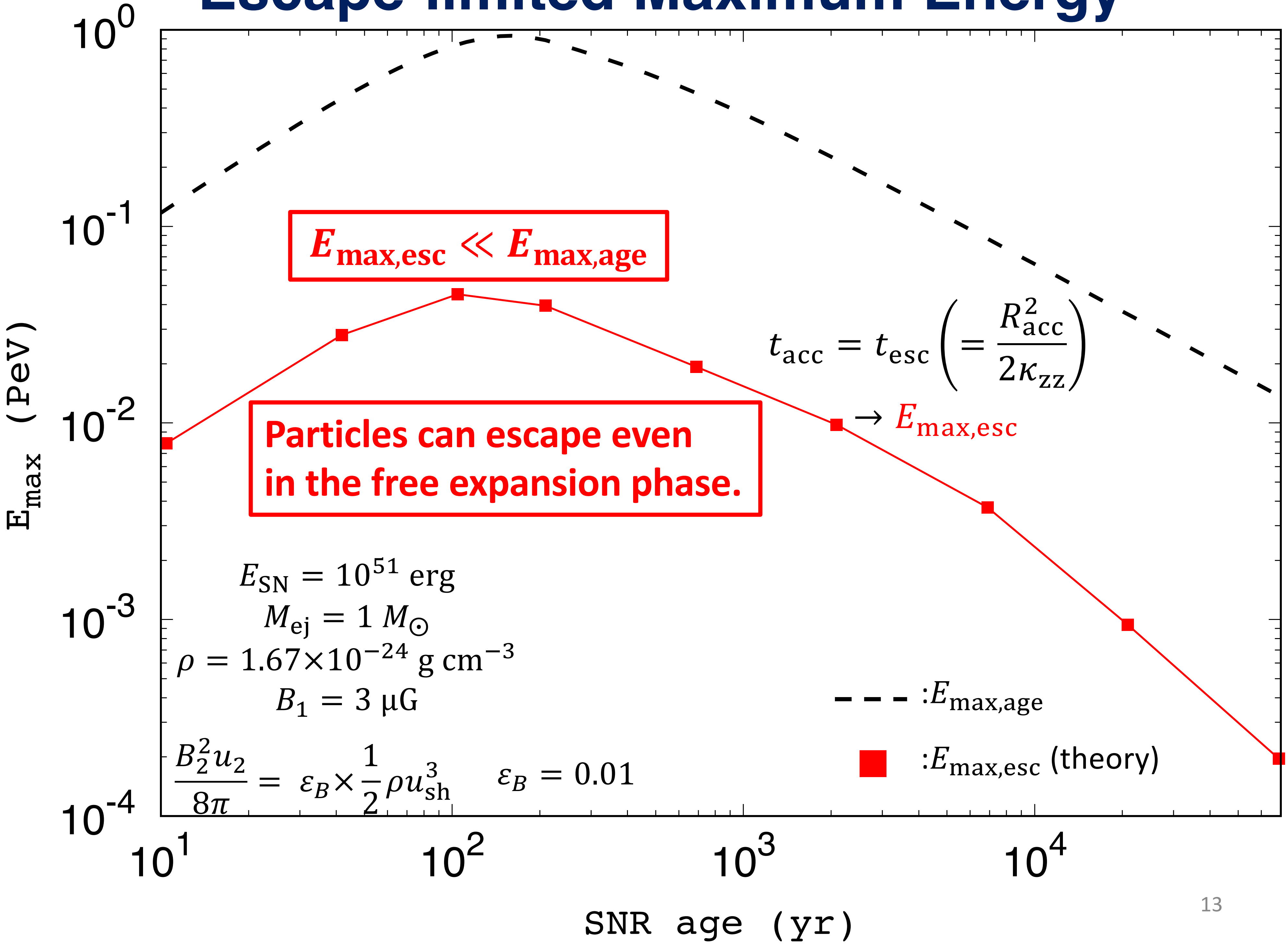


- **The size of the acceleration region is larger than the superluminal shock region.**
- **The acceleration region occupies about 20% area of the shock surface in the free expansion phase.**

Escape-limited Maximum Energy

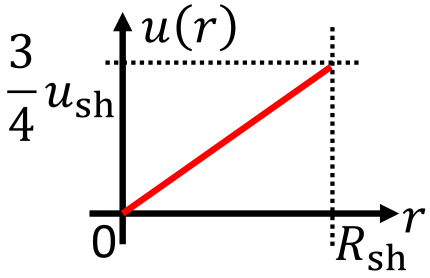


Escape-limited Maximum Energy



Simulation Setup

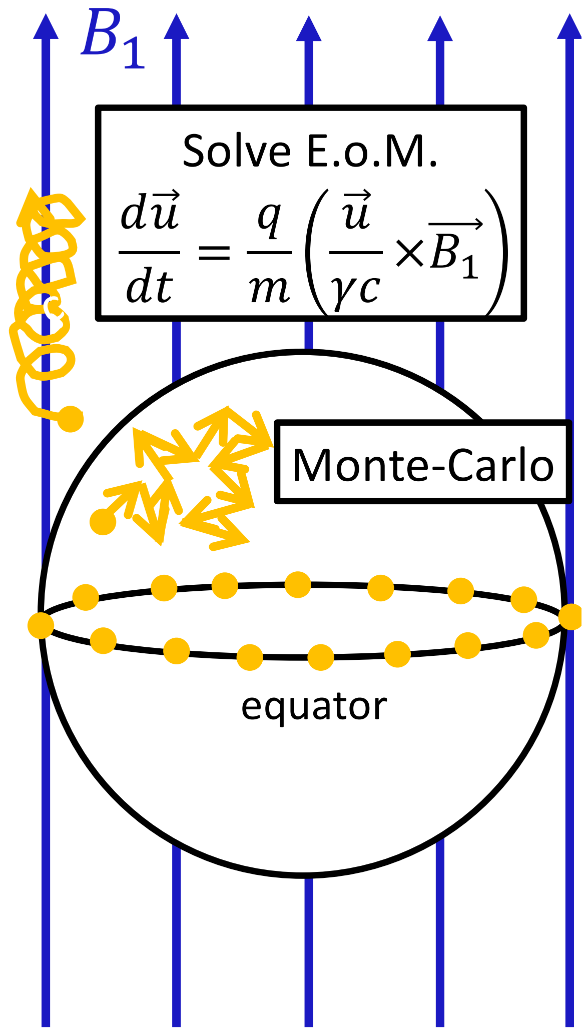
- test particle simulation (up) + Monte-Carlo (down)
- Time evolution of shock: Mckee & Truelove 1995
  ($E_{\rm SN} = 10^{51}$ erg, $M_{\rm ej} = 1\ M_\odot$, $\rho = 1.67\times10^{-24}$ g cm^{-3})
- downstream flow velocity
  (only radial component)



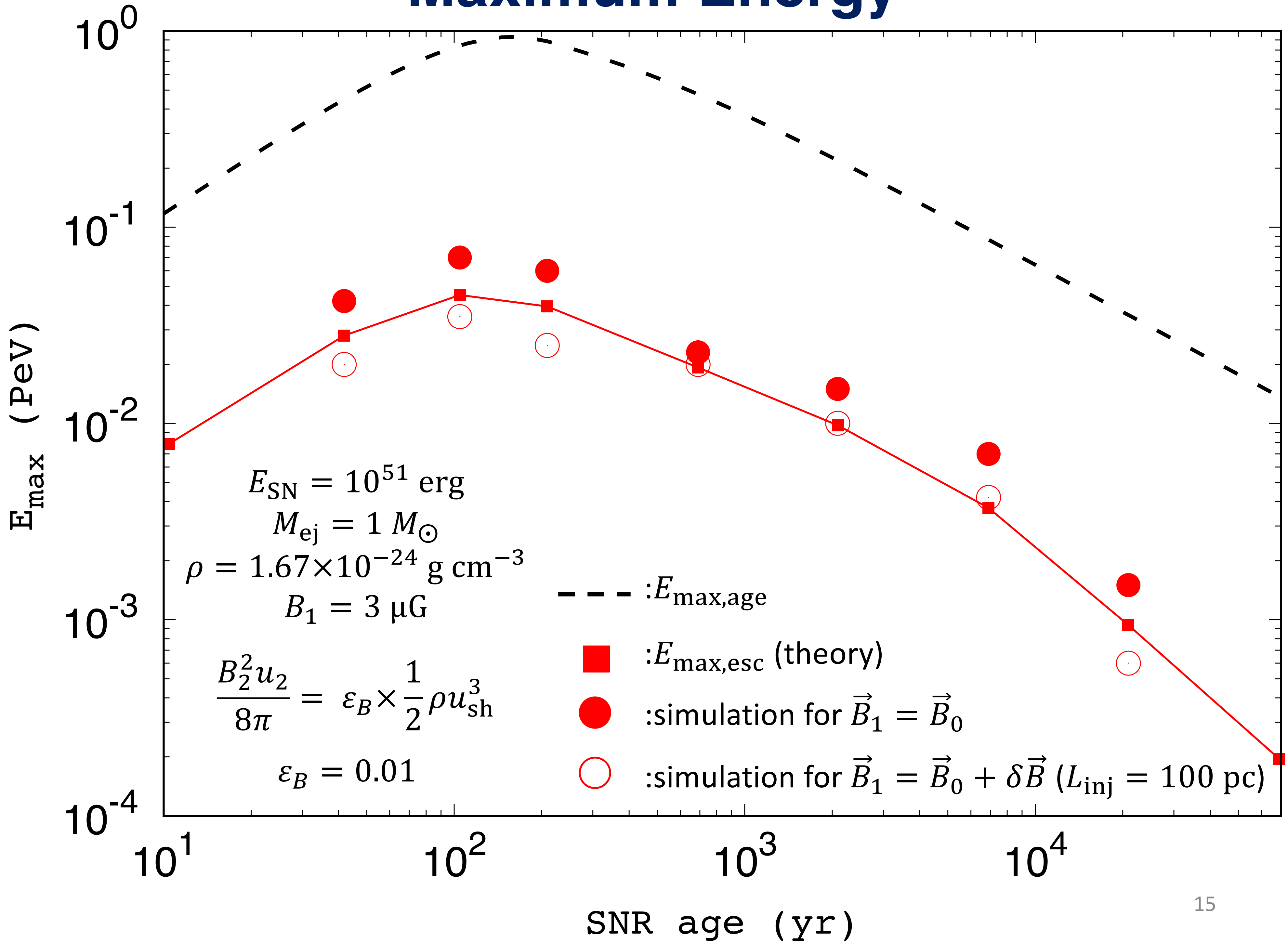
- Downstream particles are scattered in the local downstream fluid rest frame.
- downstream B field $\dfrac{B_2^2 u_2}{8\pi} = \varepsilon_B \times \dfrac{1}{2}\rho u_1^3 \qquad \varepsilon_B = 0.01$
- impulsive injection at the initial time (isotropic velocity distribution)
  ($E_{\rm inj} = 1$ TeV@$t_{\rm inj} = 40, 100, 200, 660, 2000, 6600$ yr
  $E_{\rm inj} = 100$ GeV@$t_{\rm inj} = 20000$ yr)
- upstream B field B_1 (2 types)
  - only uniform B field, $\vec{B}_0$ ($B_0 = 3\mu$G)
  - uniform B field, $\vec{B}_0 +$ fluctuation $\delta\vec{B}$

    isotropic Kolmogorov spectrum, $\sqrt{\Sigma_n \delta B^2(k_n)}/B_0 = 1$

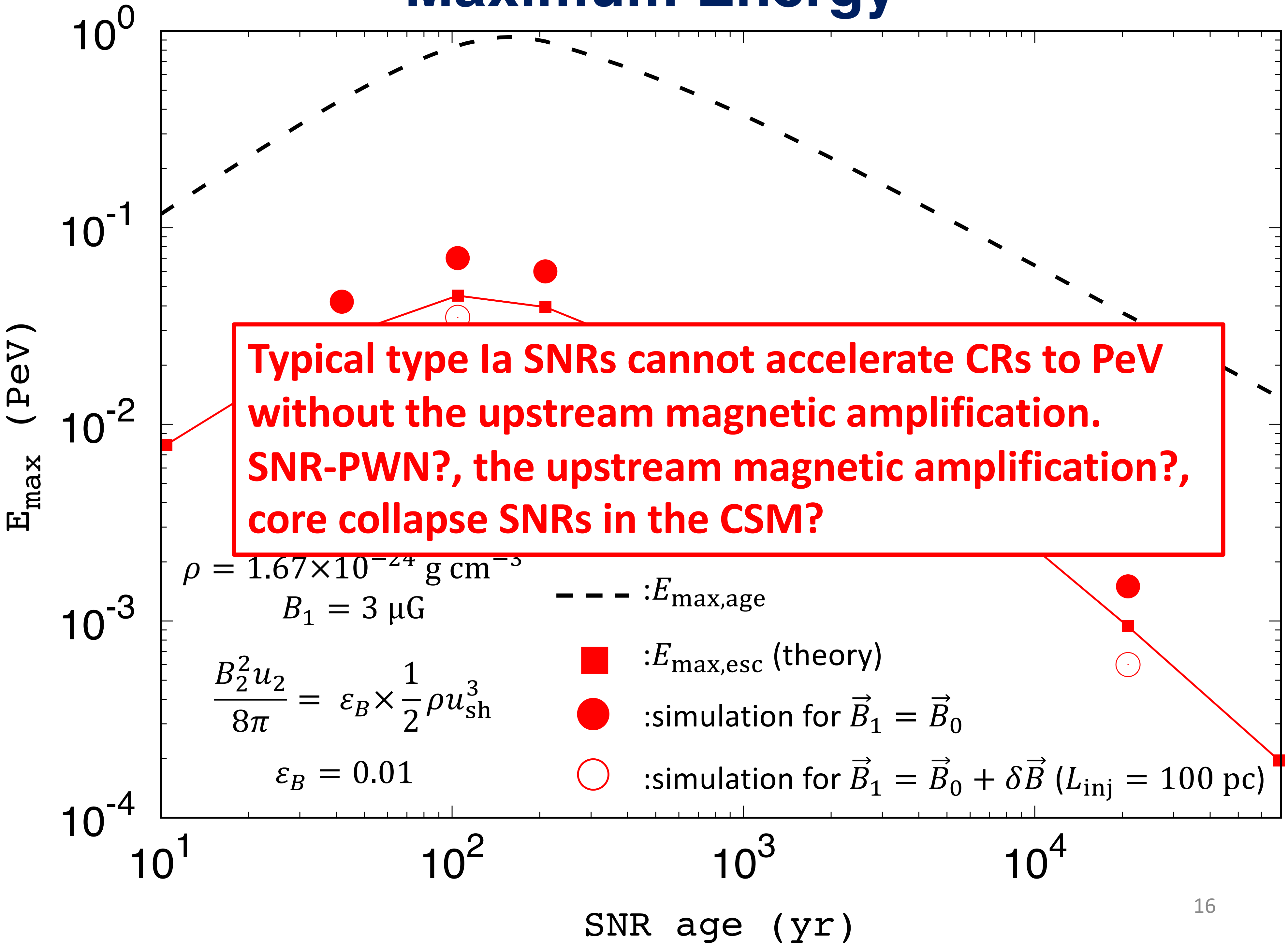
    injection scale: $L_{\rm inj} = 100$ pc
- injection position: **equator** (for $\vec{B}_1 = \vec{B}_0$), **whole sphere** ($\vec{B}_1 = \vec{B}_0 + \delta\vec{B}$)



Maximum Energy



Maximum Energy



Typical type Ia SNRs cannot accelerate CRs to PeV without the upstream magnetic amplification.
SNR-PWN?, the upstream magnetic amplification?, core collapse SNRs in the CSM?

Summary & Future Work

- Summary
  - We investigated the escape process from the perpendicular shock region of SNRs in the ISM magnetic field.
  - Particles injected to the equatorial plane escape from acceleration region while diffusing along the upstream magnetic field line.
  - the size of the acceleration region is larger than the superluminal shock region because of the finite pitch angle of particles.
  - The escape-limited maximum energy is much smaller than PeV.
    → **Type Ia SNRs in the ISM cannot accelerate CRs to PeV without the upstream magnetic amplification.**
- Future Work
  - We will investigate the case of core collapse SNRs in the CSM.

    → The upstream magnetic field structure is the shape of the Parker spiral structure.

    → Wide perpendicular shock region?