Application of parabolic equation methods to in-ice radiowave propagation for ultra high energy neutrino detection experiments

ONLINE ICRC 2021 THE ASTROPARTICLE PHYSICS CONFERENCE Berlin | Germany 37th International Cosmic Ray Conference 12–23 July 2021

RADAR ECHO TELESCOPE

Cade Sbrocco and Steven Prohira for the Radar Echo Telescope collaboration

Introduction

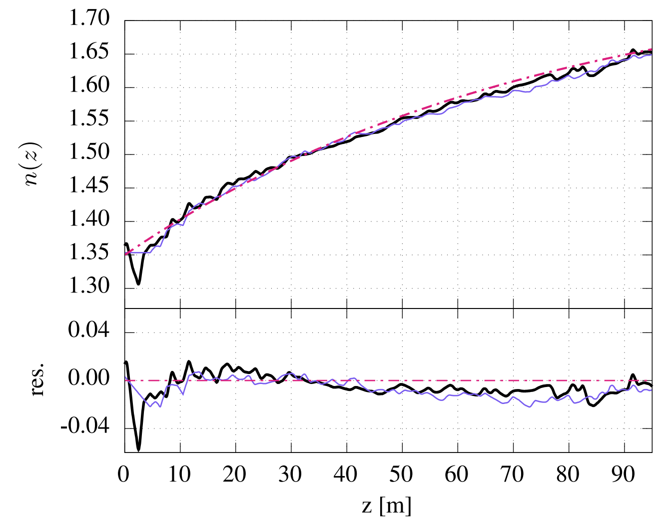
- Parabolic equation (PE) methods approximate the solution to the wave equation by only calculating the field propagating in one direction.
- PE methods approximate the solution $u(x+\Delta x)$ at range $x+\Delta x$ to be exclusively dependent upon the solution $u(x)$.

$$u(x+\Delta x) = e^{ik_0(x+\Delta x)(-1+Q)} = e^{ik_0\Delta x(-1+Q)}u(x)$$

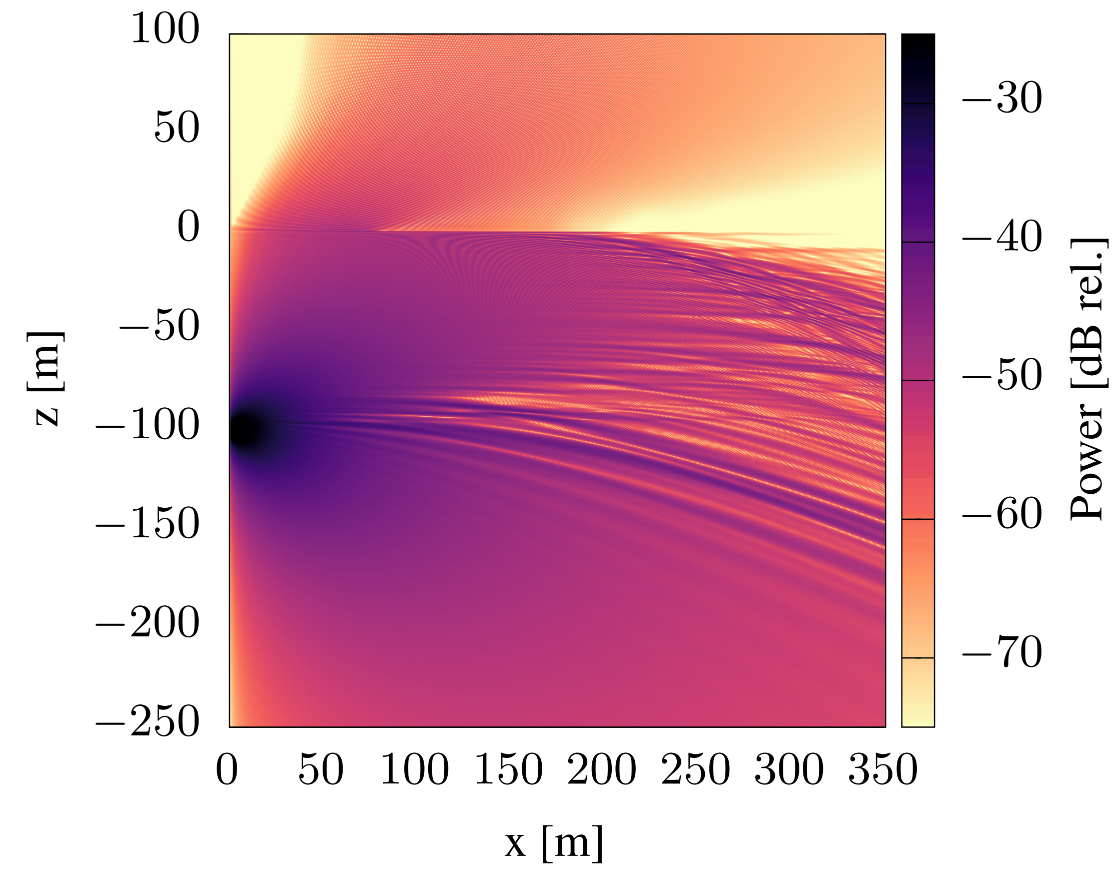
- The operator Q contains the diffractive and refractive physics governing propagation. The in-ice PE approximation has a form of Q that results in decent agreement with 'exact' finite-difference time-domain (FDTD) solutions, at a fraction of the computational cost.)

Ice properties

The top part of an ice sheet, where snow is being compacted into ice, is called the firn. This process causes a density gradient that results in a non-uniform index of refraction. This results in complicated propagation, as can be observed in the figure below, where the field focuses and de-focuses with range.

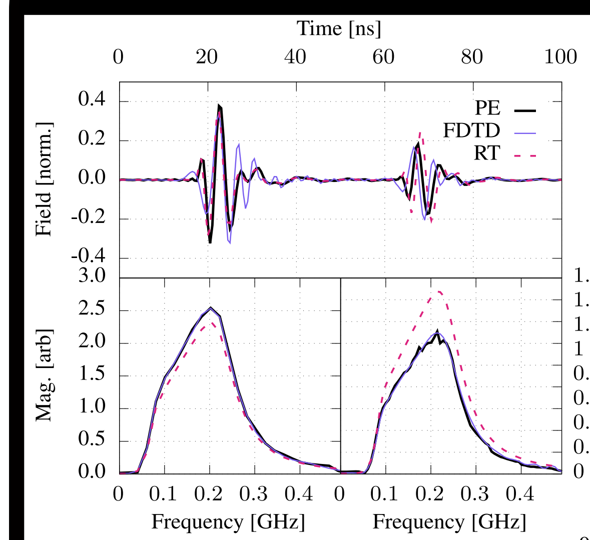


The index of refraction as a function of depth from SPICE core data, with an overlaid functional fit. Residuals shown below.



An example of in-ice radiowave propagation from a continuous source at -100m using the parabolic method.

Results

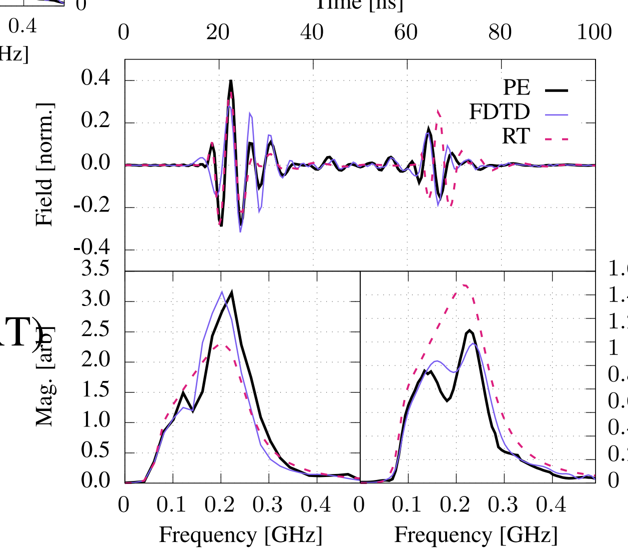


Comparison between simulations for monotonically varying functional form for n(z).

The FDTD (exact) and PE methods are compared to a geometric optics simulation, ray tracing (RT).

he same comparison, now incorporating data-driven density fuctuations.

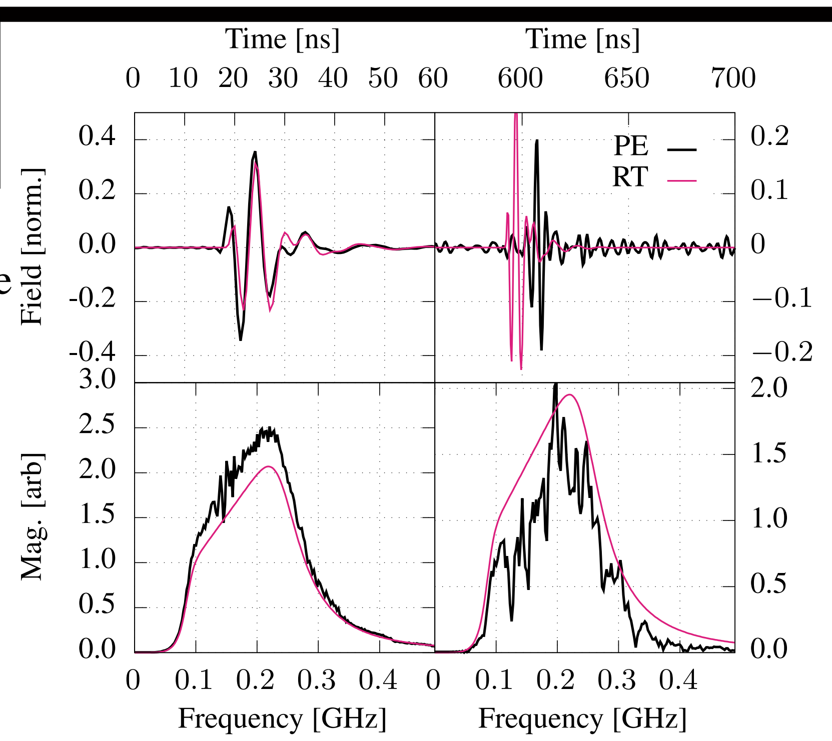
Here we see that the geometric optics method (RT) begins to disagree with the more exact solutions.



Results, cont'd

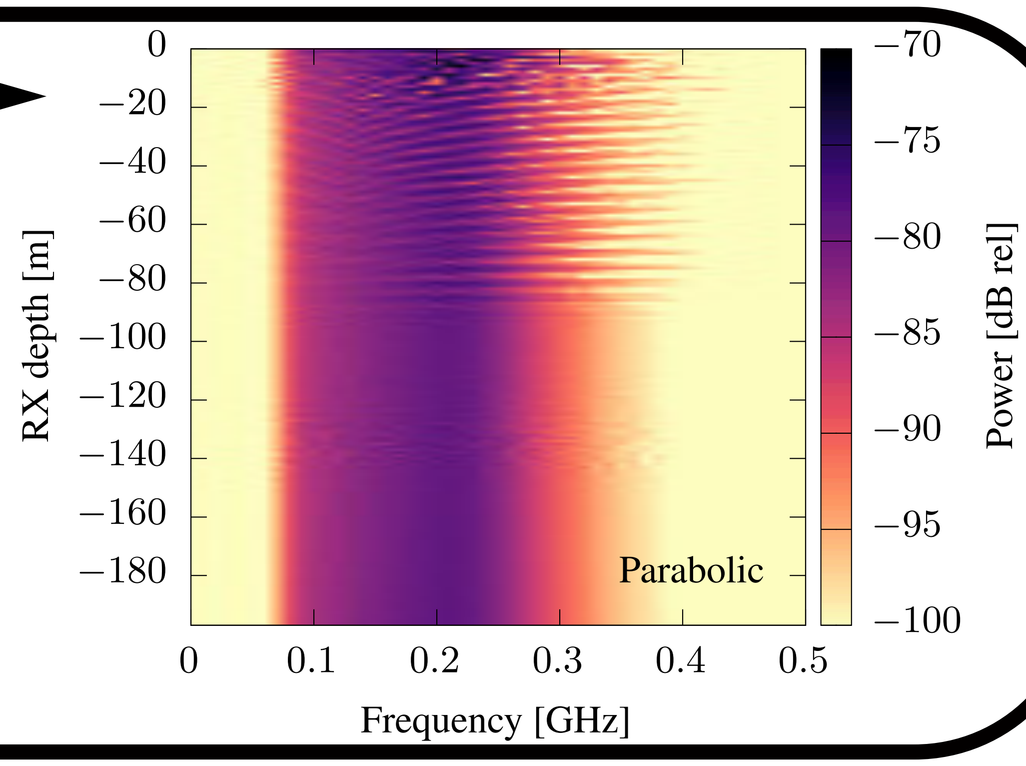
Here PE is compared to ray tracing along a long baseline. We note some diecrapencies in timing, as well as differences in the received spectra.

These are being studied to understand if they are part of the simulation, or true propagation effects.



Here we plot the received spectrum (x axis) for various receiver (RX) depths (y axis). The transmitter is 1km away and 1km deep, and each row here represents the spectrum received for a receiver at that depth.

The spectrum changes noticeably with depth in the firn.



References

- J.A Fleck et. al, Appl. Phys.10, 129 (1976)
- F.D Tappert, Springer 1977
- M Feit et. al, Appl. Opt.17, 3990 (1978)
- D.J. Thomson et. al, J. Acoust. Soc. Am.74, 1848 (1983)
- D.A. Winski et. al, Clim. Past 15 (2019)
- SP and CS et.al, Phys. Rev. D 103, 103007
- https://github.com/prchyr/paraPropPython

Air/Acoustic PE

SPICE core

in-ice PE