A Kinetic Study of the Saturation of the Bell Instability

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Motivation

The acceleration of cosmic rays (CR) at astrophysical shocks such as Supernova Remnants (SNR) requires the presence of strong magnetic fields that confine them close to the shock. Understanding how these fields are generated is still an open question. It is thought that the **non-resonant cosmic ray-driven** instability, also known as the **Bell instability**, is capable of rapidly amplifying small magnetic field perturbations. Although it has been previously studied [4, 5, 3] with PIC simulations, the saturation of the Bell instability is not completely understood to this day.

The Problem

We study the Bell instability using *dHybridR* [2] simulations, with kinetic ions and fluid electrons, by injecting CRs in the simulation box at a constant rate. We study the conditions that lead to the saturation of the Bell instability's growth and derive a formula to predict the level of the final magnetic field as a function of the initial conditions in our simulations. To do that, we have run a large number of simulations and surveyed a range of parameters that characterize the CR population.

Basic Concepts

In our work, the Bell instability grows when CR stream parallel to an ambient magnetic field B_0 in the $\hat{\mathbf{x}}$ (parallel) direction and, as a result, small magnetic field perturbations, δB, in the perpendicular direction are amplified. The fastest growing mode [1, 6] is

$$k_{\max} = \frac{4\pi J_{\rm cr}}{c\ B_0} = \frac{1}{2}\left(\frac{n_{\rm cr}}{n_g}\right)\left(\frac{v_{\rm cr}}{v_{\rm A,0}}\right) d_i^{-1} \ ,$$

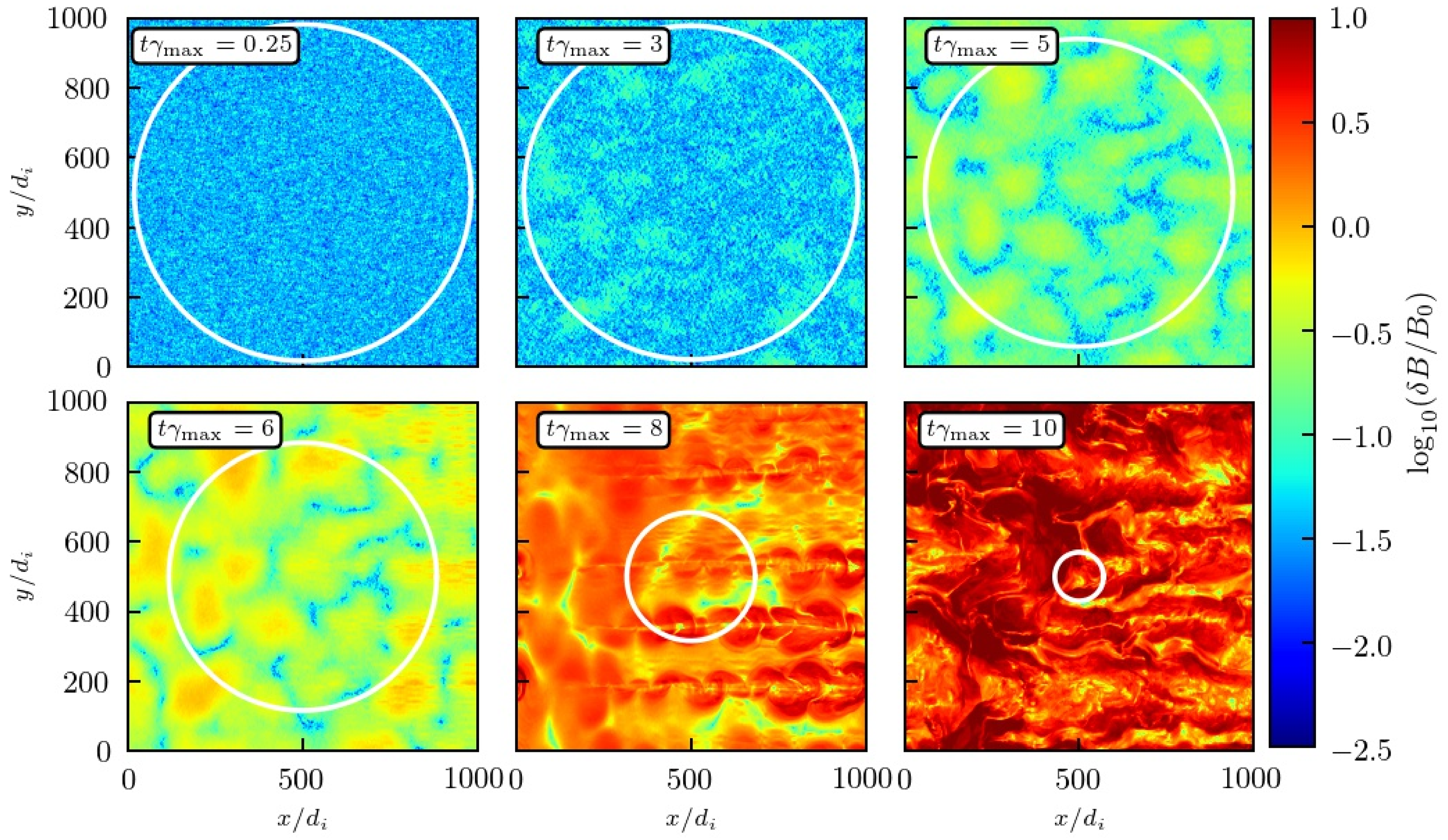
where $v_{A,0} = B_0/\sqrt{4\pi m n_g}$ is the Alfvén speed based on the background plasma density n_g and magnetic field B_0, c is the speed of light, $d_i = v_{A,0}/\Omega_{ci}$ is the ion inertial length, $\Omega_{ci} = eB_0/(mc)$ is the ion gyrofrequency and m is the ion mass; its growth rate is found to be

$$\gamma_{\max} = k_{\max} v_{\rm A,0} = \frac{1}{2}\left(\frac{n_{\rm cr}}{n_g}\right)\left(\frac{v_{\rm cr}}{v_{\rm A,0}}\right)\Omega_{ci} \ .$$

The magnetic field perturbations are amplified due to the presence of the current, $J_{\rm cr} = e n_{\rm cr} v_{\rm cr}$, carried by the CR particles of number density $n_{\rm cr}$ moving with average velocity $v_{\rm cr}$.

The *dHybridR* Simulations

In our simulations the ions are treated as macro-particles governed by the relativistic Lorentz force law and the electrons are a massless, charge neutralizing fluid. CRs are injected at a constant rate in our box from the left boundary such that the bulk motion of CRs is along B_0. We simulate 2D boxes and consider all 3 components of the fields, thus making our simulations 2.5D. The plot shows 6 time frames of the simulation where the strength of the perpendicular magnetic field is illustrated by the color bar. The Larmor radius of the CR particles in the total magnetic file is also shown as a white circle. For comparison, the wavelength of the fastest growing mode in the simulation box is $\lambda_{\max} \approx 125\ d_i$.

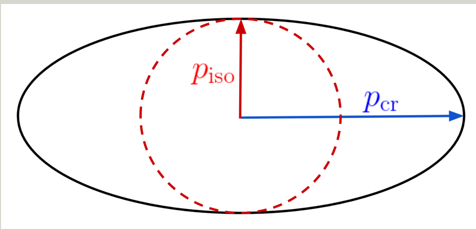


- We see an initial rapid growth of $\delta B/B_0 \sim e^{\gamma_{\max} t}$ with time until $t \sim 8\gamma_{\max}^{-1}$.
- Structures of strong magnetic field form in the simulation box which increase in size.
- When $\delta B/B_0$ reaches unity, CR particles get scattered off the magnetic field structures.
- This leads to transfer of momentum from the CRs to the plasma and couples the two species
- The saturation of the magnetic field's growth happens due to the coupling which reduces the CR current in the plasma's frame of reference.

Results

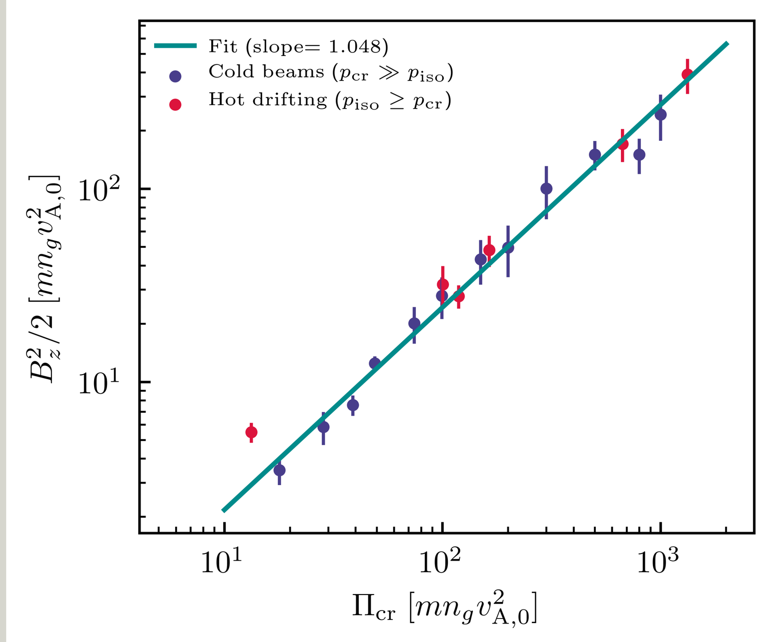
The saturated magnetic field for various simulations is shown in the plot as a function of the anisotropic CR momentum flux density, $\Pi_{\rm cr}$, in the $x-$direction in which the CRs drift. This plot implies that at saturation of the Bell instability:

$$\frac{B_z^2}{2} \approx \Pi_{\rm cr} \approx n_{\rm cr}\gamma_{\rm iso}\left[1 + \left(\frac{p_{\rm iso}}{\gamma_{\rm iso} mc}\right)^{2/3}\right]\frac{p_{\rm cr}^2}{\gamma_{\rm cr}}$$



We conclude that:

- We can calculate the final, amplified magnetic field based only on the input CR parameters.
- The quantities that determine the magnetic field's final strength are: the CR drift and isotropic momentum $p_{\rm cr}$ and $p_{\rm iso}$, where their corresponding Lorentz factors are $\gamma_{\rm cr}$ and $\gamma_{\rm iso}$, and the CR number density $n_{\rm cr}$.



References

[1] A. R. Bell. Turbulent amplification of magnetic field and diffusive shock acceleration of cosmic rays. *MNRAS*, 353:550–558, September 2004.

[2] C. C. Haggerty and D. Caprioli. dHybridR: A Hybrid Particle-in-cell Code Including Relativistic Ion Dynamics. *ApJ*, 887(2):165, 12 2019.

[3] Y. Ohira, T. Terasawa, and F. Takahara. Plasma Instabilities as a Result of Charge Exchange in the Downstream Region of Supernova Remnant Shocks. *The Astrophysical Journal Letters*, 703(1):L59, 2009.

[4] M. A. Riquelme and A. Spitkovsky. Nonlinear Study of Bell's Cosmic Ray Current-Driven Instability. *ApJ*, 694(1):626–642, March 2009.

[5] T. Stroman, M. Pohl, and J. Niemiec. Kinetic Simulations of Turbulent Magnetic-Field Growth by Streaming Cosmic Rays. *ApJ*, 706:38–44, November 2009.

[6] V. N. Zirakashvili, V. S. Ptuskin, and H. J. Völk. Modeling Bell's Nonresonant Cosmic-Ray Instability. *The Astrophysical Journal*, 678(1):255, 2008.