Seasonal variation of atmospheric muons

A theoretical discussion with emphasis on compact underground detectors like MINOS where detector dimensions are small compared to their depths

Outline

- Muon production vs atmospheric slant depth
  - Analytic solutions of hadronic cascade equations
  - Parameterization based on simulations (see Stef Verpoest, PoS 357)
- Effective temperature, T_{eff}
  - Convolution of T(X) with $P_{\mu}(X)$
  - Convolution of T(X) with $dP_{\mu}(X)/dT$
- Correlation of muon rates with T_{eff}
  - Compare two energy/depth ranges
  - MINOS at FNAL ($E\mu \approx 100$ GeV); at Soudan (≈TeV)

Muon production and rates

$$R(\theta, \phi) = \int dX \int_{E_{\mu,min}} dE_\mu \, A_{\rm eff}(E_\mu, \theta, \phi) P(E_\mu, \theta, X)$$

For an extended detector like IceCube, muon survival depends on depth of entry and is included in the dependence of Aeff on Eμ
For a compact detector, muon survival depends only on slant depth between the surface and the detector, and detector area factors out*:

$$\begin{aligned} R(\theta) &= A_{\rm eff}(\theta) \int dX \int_{E_{\mu,min}} dE_\mu \, P(E_\mu, \theta, X) \\ &= A_{\rm eff}(\theta) \int dX P(> E_{\mu,min}, \theta, X) \\ &= A_{\rm eff}(\theta) \, I(E_{\mu,min}, \theta), \end{aligned}$$

*assuming a flat overburden with azimuth-averaged detector area

Analytic: P(>E_μ,θ,X)

$$P(> E_{\mu,min}, \theta, X) = F(E_\mu) \frac{A_{\pi\mu}(X)}{\gamma + (\gamma+1) B_{\pi\mu}(X) E_\mu \cos\theta / \epsilon_\pi}$$

$$A_{\pi\mu}(X) = \frac{Z_{N\pi}}{\lambda_N(\gamma+1)} \frac{1 - r_\pi^{\gamma+1}}{1 - r_\pi} e^{-X/\Lambda_N},$$

$$B_{\pi\mu}(X) = \frac{\gamma+2}{\gamma+1} \frac{1 - r_\pi^{\gamma+1}}{1 - r_\pi^{\gamma+2}} \frac{X}{\Lambda^*} \frac{e^{-X/\Lambda_N}}{e^{-X/\Lambda_\pi} - e^{-X/\Lambda_N}},$$

Equations derived from two channel solution to the hadronic cascade equation
Similar forms for the $K^\pm$ channel are added to get the full P(>E_μ,θ,X)
Integration over X gives *inclusive* spectrum of muons, not applicable to multi-μ

Parameterization for P(>E_μ,θ,X)

$$\begin{aligned}\frac{\mathrm{d}N}{\mathrm{d}X}(>E_\mu, E_0, A, \theta, X) &= N_{\max} \times \exp((X_{\max}-X)/\lambda) \qquad (4)\\ &\times \left(\frac{X-X_0}{X_{\max}-X_0}\right)^{(X_{\max}-X_0)/\lambda} \times \frac{X_{\max}-X}{\lambda(X-X_0)}\\ &\times F(E, E_\mu, \theta, X) \times \frac{1}{f E_\mu \cos\theta X} \times \left(1-\frac{AE_\mu}{E_0}\right)^{\alpha_2}.\end{aligned}$$

First 2 lines represent production of parent mesons; Factors in last line are:
F for decay vs re-interaction, energy-angular dependence, threshold factor.
A is nuclear mass; E_0 total energy. F contains the critical energies for $\pi^\pm$ and $K^\pm$

$$\epsilon_\pi = \frac{m_\pi c^2}{c\tau_\pi}\frac{RT}{Mg} \approx 115\,\mathrm{GeV} \times \frac{T}{220\,\mathrm{K}} \qquad \epsilon_K = 857\,GeV \times \frac{T}{220K}$$

Parameterization applied to seasonal dependence of multi-μ in arXiv: 2106.12247
Here applied to total rates, which are dominated by single muons

Effective temperature

- Rates depend on T-dependence of $\epsilon_\pi, \ \epsilon_K$
- T_{eff} is a single value for each time interval
  - Option 1 $T_{\rm eff}(\theta) = \frac{\int {\rm d}X\, P(E_\mu, \theta, X)\, T(X)}{\int {\rm d}X\, P(E_\mu, \theta, X)}$
  - Option 2: calculate the variance w.r.t. T of the rate

$$\Delta{\rm R}(\theta) = \int {\rm d}X \int {\rm d}E_\mu\, A_{\rm eff}(E_\mu, \theta) \times \frac{{\rm d}P(E_\mu, \theta, X)}{{\rm d}T} \Delta T.$$

  - Then set ΔR=0 and $\Delta T = T(X) - T_{\rm eff}$ to get

$$T_{\rm eff}(\theta) = \frac{\int {\rm d}X \int {\rm d}E_\mu\, A_{\rm eff}(E_\mu, \theta) T(X) \frac{{\rm d}P(E_\mu,\theta,X)}{{\rm d}T}}{\int {\rm d}X \int {\rm d}E_\mu\, A_{\rm eff}(E_\mu, \theta) \frac{{\rm d}P(E_\mu,\theta,X)}{{\rm d}T}}.$$

Correlation of rates with T_{eff}

$$\frac{\delta R}{R_{av}} = \alpha_T \frac{\delta T}{T_{av}}$$

This equation defines the correlation between rate and T_{eff} with

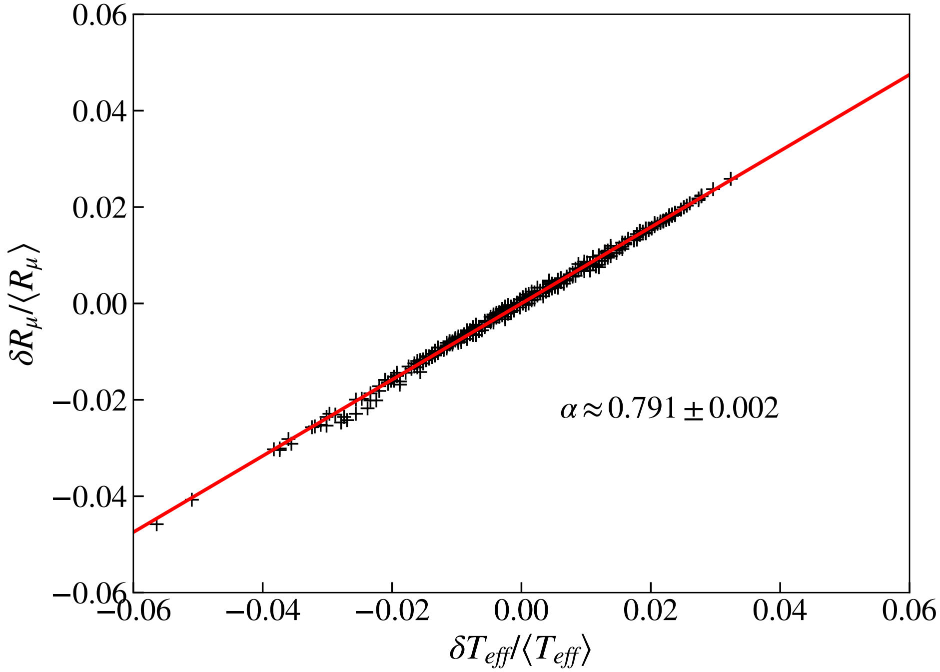
$$\delta R = R_i - R_{av} \text{ and } \delta T = T_{\mathrm{eff},i} - T_{\mathrm{eff},av}$$

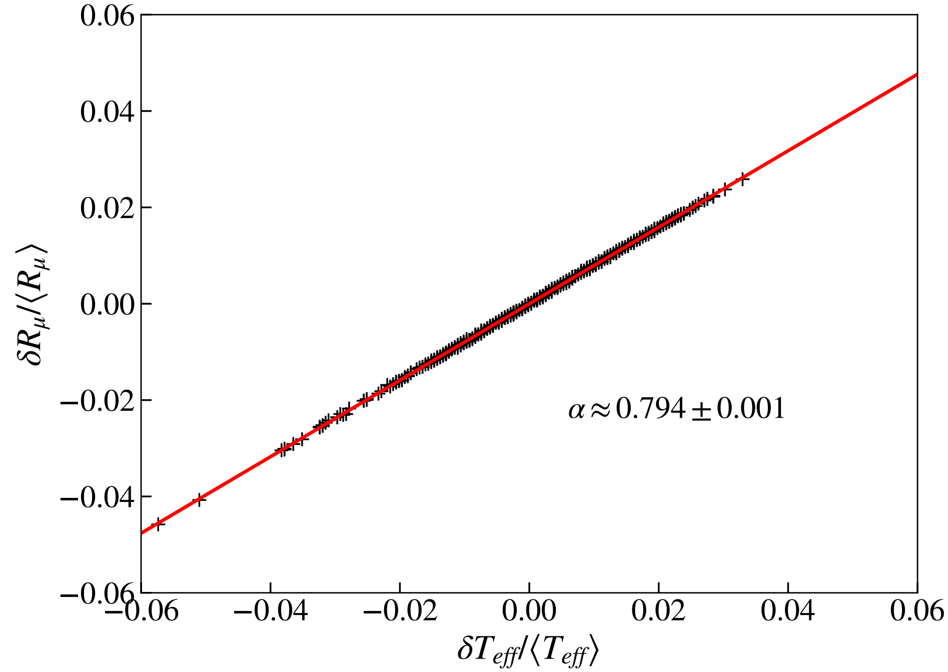
where the subscript *i* indicate a day or other time interval for which rates and T_{eff} are available.

Results of calculations

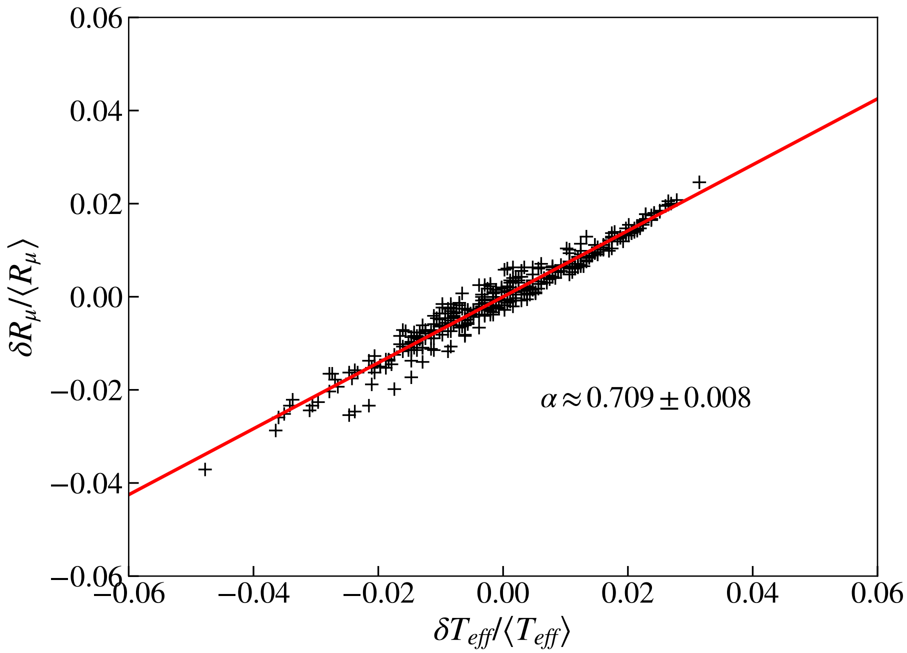
- The following sequence of plots shows the correlation between calculated rate and T_{eff}
  - Left: linear definition of T_{eff}
  - Right: derivative definition
- Results for both the TeV region (MINOS FD) and the lower-energy region of MINOS ND
  - The headings give the calculation type: analytic or parameterization

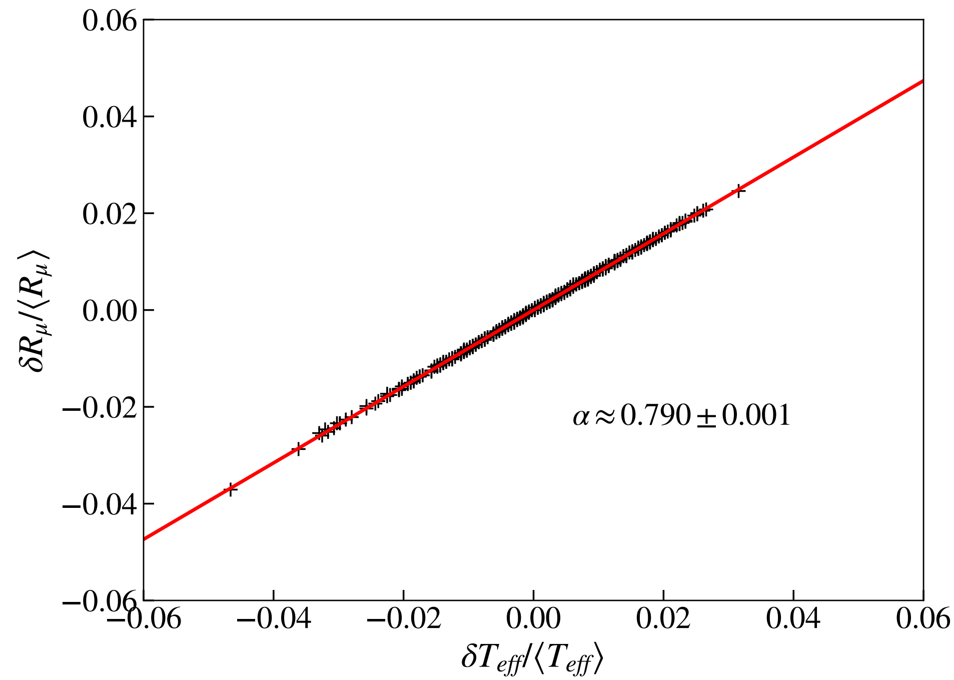
TeV range (FD) analytic



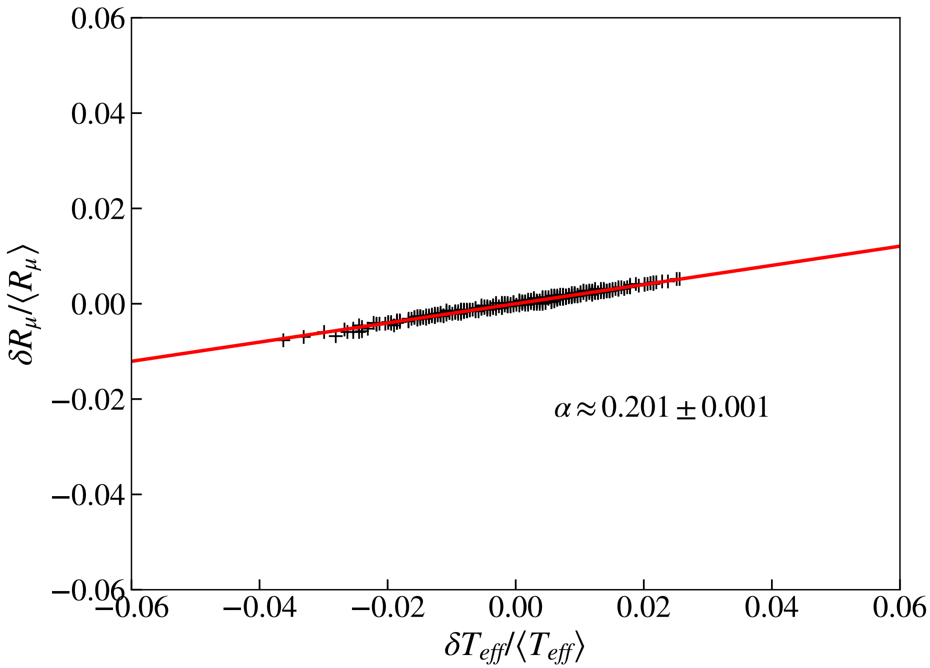


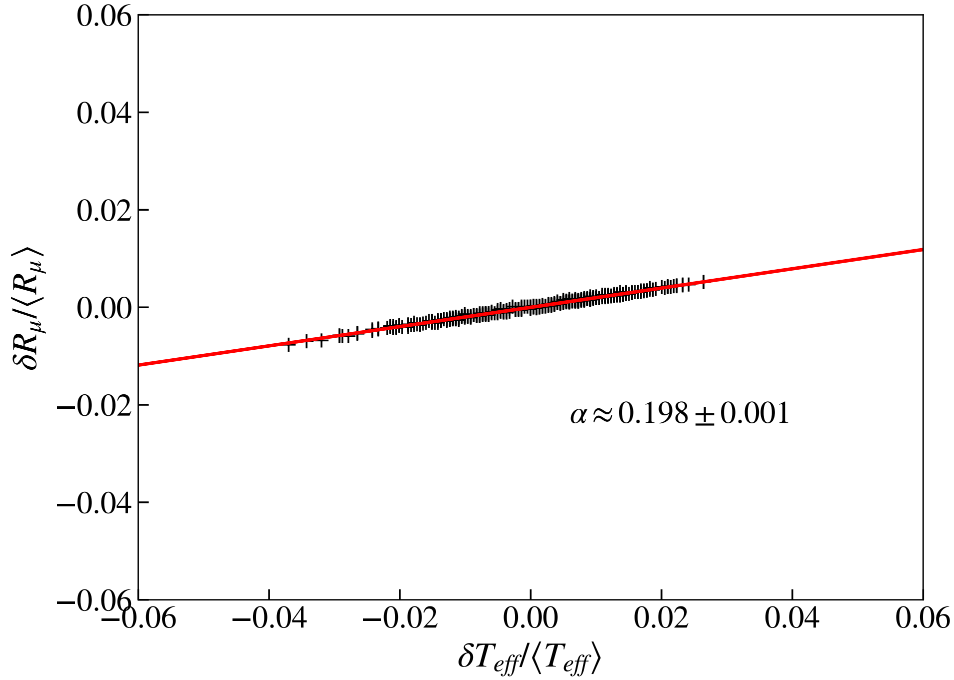
TeV range (ND) parameterization



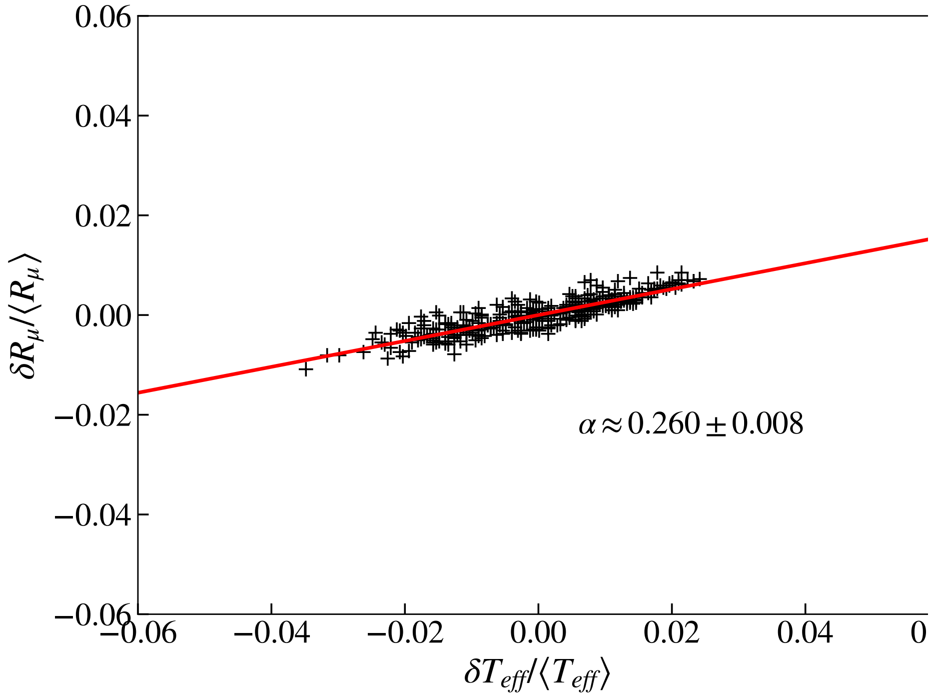


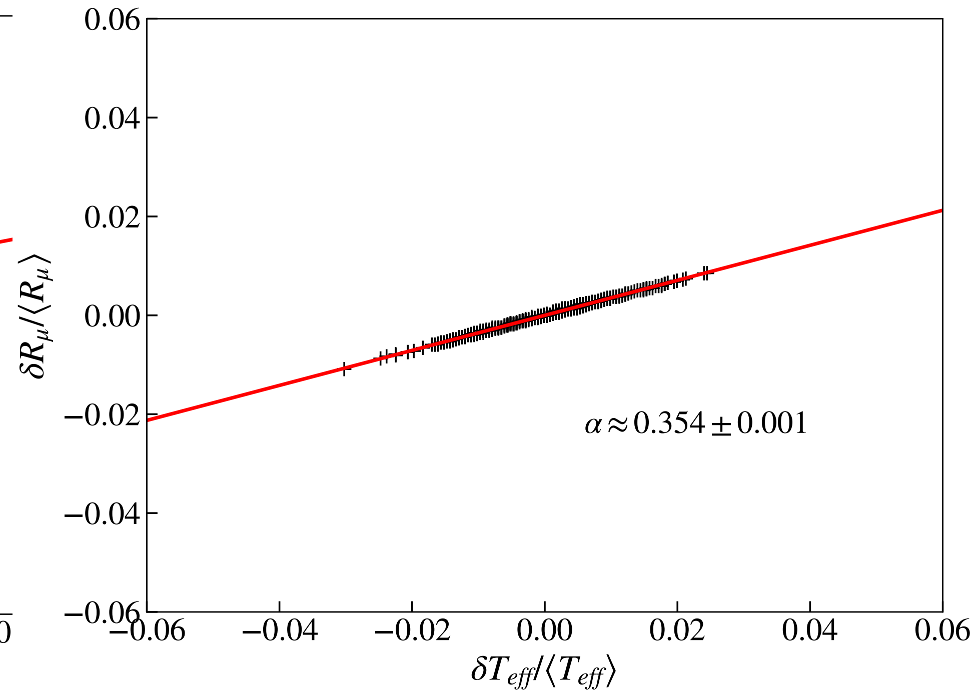
Low energy (ND) analytic





Low energy (ND) parameterization





Conclusions

- The derivative definition of T_{eff}
  - Reduces scatter; minimizes deviation of calculated rate from value expected for a given T_{eff}
- Correlation with T_{eff} is larger at high energy
  - Expected because pions are fully correlated at TeV but not for the lower energy where $E_\mu \approx \varepsilon_\pi$
  - The pion channel dominates the rate
- The $K^\pm$ fraction is larger for the parameterization than for the analytic calculations, but, for the low-energy case, the α_T are also larger
  - contrary to expectation because the partial $\alpha_{T,K}$ are smaller than the $\alpha_{T,\pi}$