Muon number rescaling in simulations of air showers

[1] Nataliia Borodai, [2] Ralph Engel, [1] Dariusz Góra (presenter), [1] Jan Pękala,
[2] Tanguy Pierog, [2] Markus Roth, [1] Jaroslaw Stasielak, [2] Michael Unger,
[2] Darko Veberic and [1] Henryk Wilczyński

[1] *Institute of Nuclear Physics PAN, Radzikowskiego 152, Cracow, Poland*

[2] *Karlsruhe Institute of Technology (KIT), Institute for Astroparticle Physics, Karlsruhe, Germany*

Outline:
- Introduction: top-down simulations
- Basic principle of z-method
- The cross-check of the z-method with MC simulations
- The beta calculation

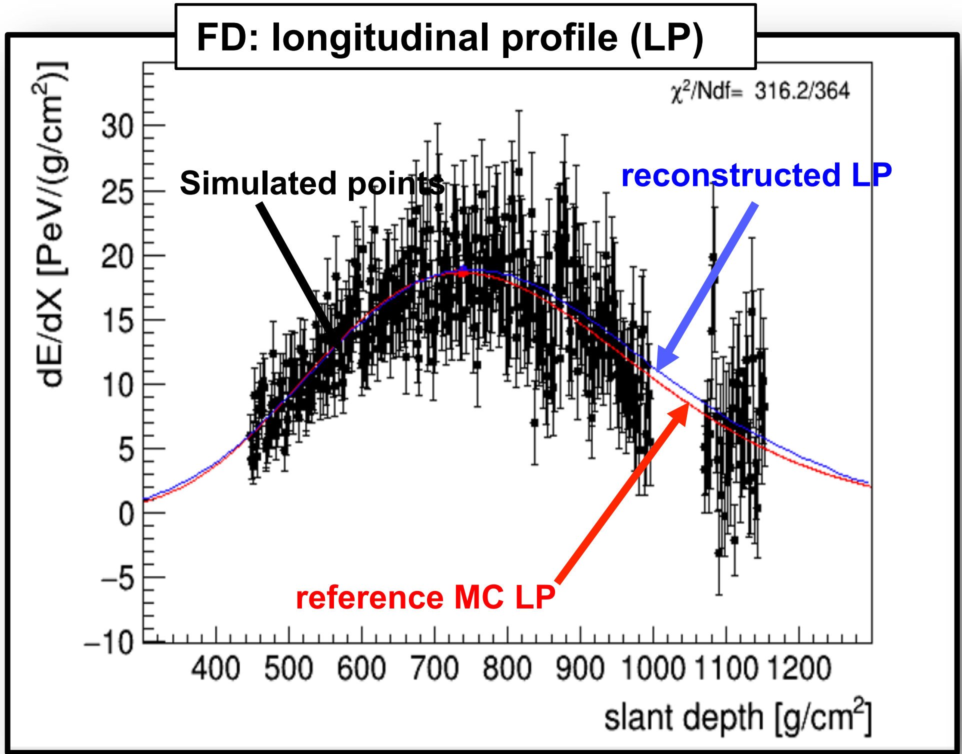
ICRC 2021- #412

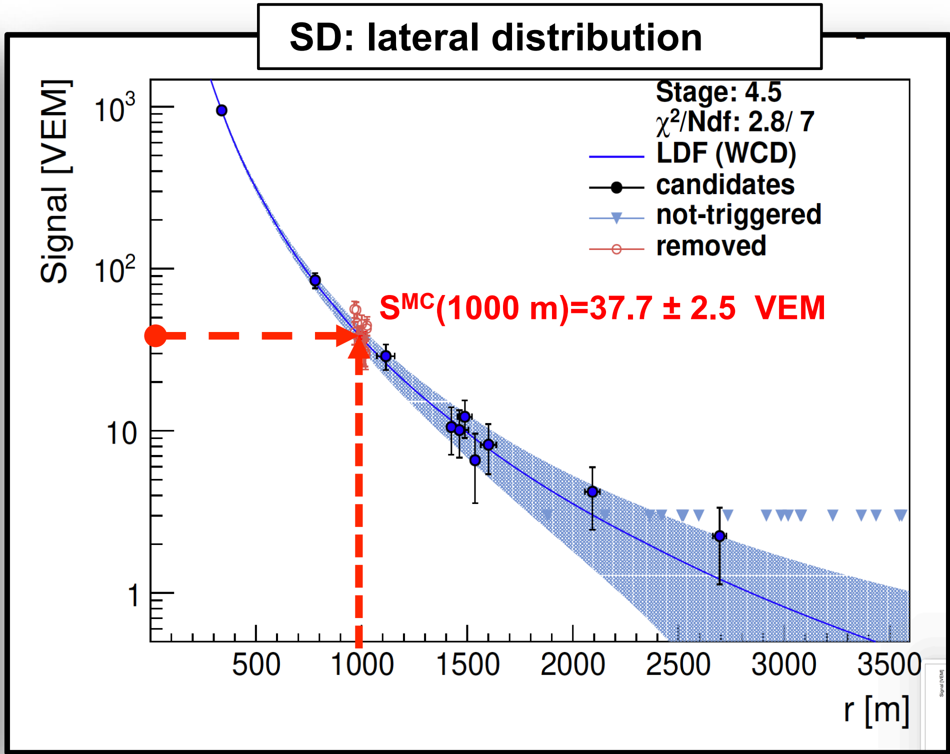
Introduction

> Simulations of extensive air showers using current hadronic interaction models predict too small number of muons. The muon number predicted by the LHC-tuned models, such as EPOS-LHC and QGSJetII-04, is 30% to 60% lower than what is observed at the shower energy of 10^{19} eV [A. Aab et al., Phys. Rev. Lett. 117,192001 (2016)]

>Top-Down method: predict signals in **the fluorescence (FD) and surface detectors (SD)** of the Pierre Auger Observatory on simulation basis.

Simulated event seen by FD and SD (hybrid event), $E_{primary}=10^{19}$ eV, proton



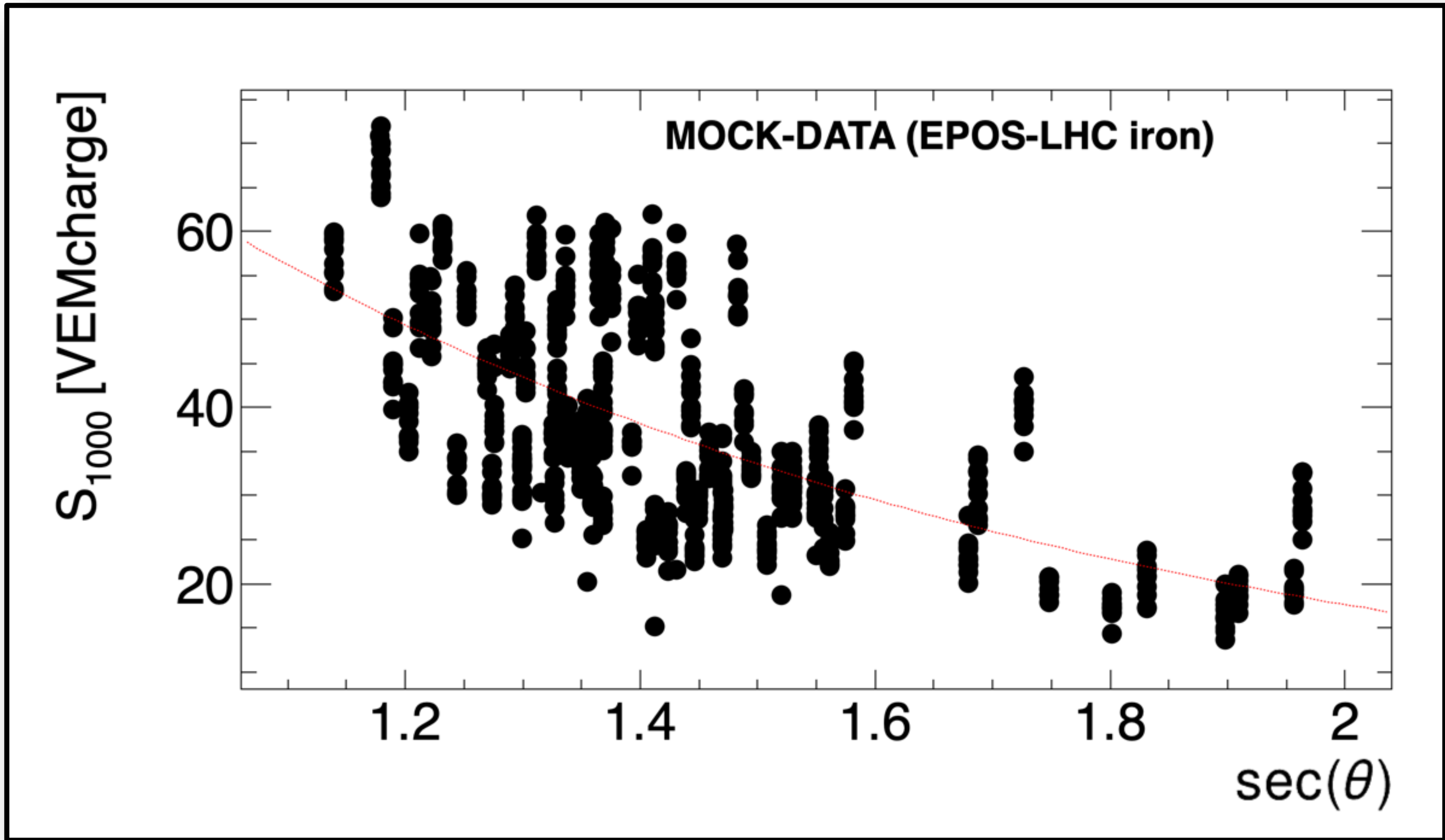


Software used: CORSIKA-75600 D. Heck et al., Report FZKA 6019 (1998), Auger Offline software S. Argiro et al., NIM in Phys. A 580, 1485 (2007)
(from 20 Offline reconstructions of the event we use 10 with the best chi² to reference profile)
Models used: EPOS-LHC, QGSJetII-04.
Primaries used: proton, helium, nitrogen, iron

for more details Cz. Porowski, PhD thesis (2019): https://rifj.ifj.edu.pl/handle/item/289

MOCK-DATA set

> In order to take into account the muon discrepancy between data and MC, we can use simulation for EPOS-LHC at 10^{19} eV as a MOCK-DATA set
… but p, He, N, Fe TD-simulations with QGSJETII-04 as Monte Carlo signal



This assumption can reproduce Auger $S^{DATA}(1000)$ quite well: Balazs Kegl for the Pierre Auger Collab.,ICRC-2013 [astro-ph 1307.5059]

Method to get the muon scaling factors

from A. Aab et. al., PRL 117, 192001 (2016)

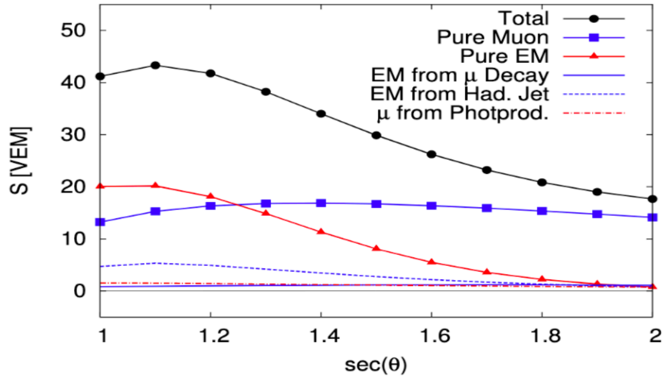


FIG. 3. The contributions of different components to the average signal as a function of zenith angle, for stations at 1 km from the shower core, in simulated 10 EeV proton air showers illustrated for QGSJet-II-04.

Results published in PRL 117, 192001 (2016) using
- **SENECA , S_{1000}, but composition of elements from FD measurements**

The main results: scaling factors shown below:

$$S_{\mathrm{resc}}(R_E, R_{\mathrm{had}})_{i,j} \equiv R_E\, S_{EM,i,j} + R_{\mathrm{had}}\, R_E^{\alpha}\, S_{\mathrm{had},i,j}. \quad (1)$$

TABLE I. R_E and R_{had} with statistical and systematic uncertainties, for QGSJet-II-04 and EPOS-LHC.

Model	R_E	R_{had}
QII-04 p	$1.09 \pm 0.08 \pm 0.09$	$1.59 \pm 0.17 \pm 0.09$
QII-04 Mixed	$1.00 \pm 0.08 \pm 0.11$	$1.61 \pm 0.18 \pm 0.11$
EPOS p	$1.04 \pm 0.08 \pm 0.08$	$1.45 \pm 0.16 \pm 0.08$
EPOS Mixed	$1.00 \pm 0.07 \pm 0.08$	$1.33 \pm 0.13 \pm 0.09$

> Instead of S_{1000} we use the difference between total signals: $z_j \equiv S_{1000,j}^{\mathrm{MOCK-DATA}} - S_{1000,j}^{\mathrm{MC}}$ as the main observable.

Rescaling factor for electromagnetic part of EAS in TD metod equals to 1

R_E=1 results also from A. Aab et. al., PRL 117, 192001 (2016)

Rescaling factor for muonic part of EAS

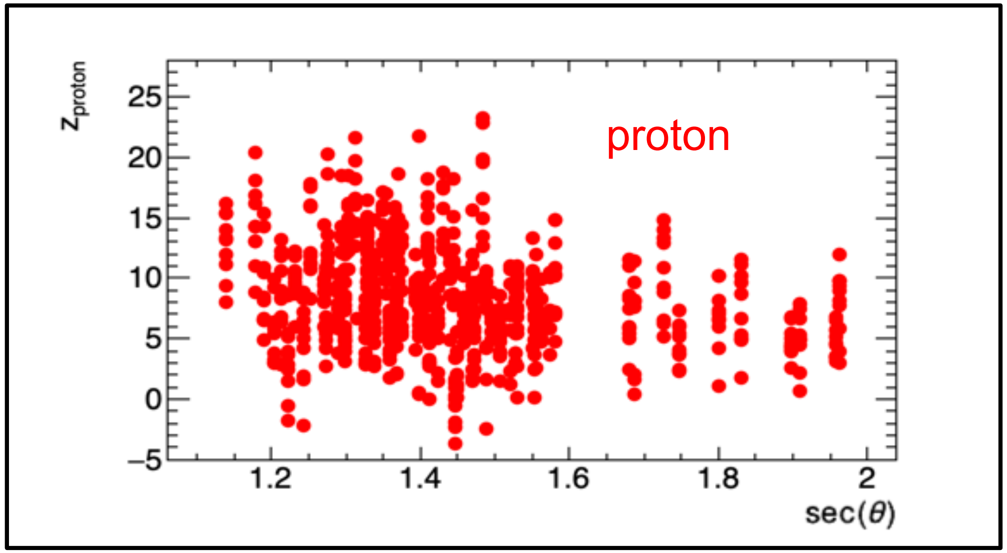
$$S_{1000}^{MOCK-DATA}(R_E, R_\mu)_{i,j} = R_E S_{EM,i,j}^{MC} + R_\mu R_E^{\alpha} S_{\mu,i,j}^{MC}$$

$i = 1 := \mathrm{proton}, i = 2 := \mathrm{helium}, i = 3 := \mathrm{nitrogen}, i = 4 := \mathrm{iron}$

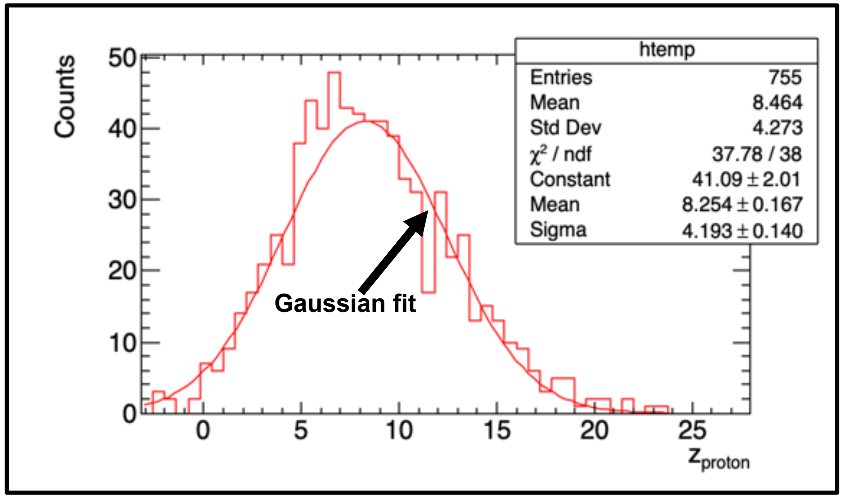
$$S_{\mu,i}^{MC} = \frac{z_i}{R_{\mu,i} - 1} \quad \text{for } R_E = 1$$

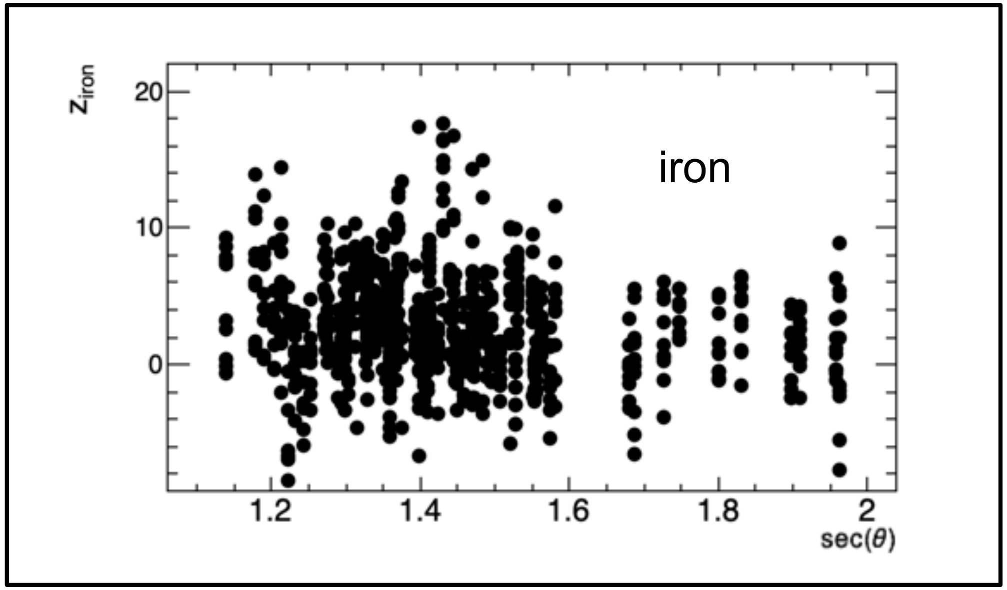
> We introduced different muon scaling factors for different primaries (p, He, N, Fe):
- average z_i is connected to average muon Monte Carlo signal at 1000 m $\langle S_{\mu,i}^{MC} \rangle$
- z_i should only slightly depend on the zenith angle

Distribution of z=S$^{MOCK-DATA}$ (1000) – S^{MC}(1000) (QGSJETII-04)

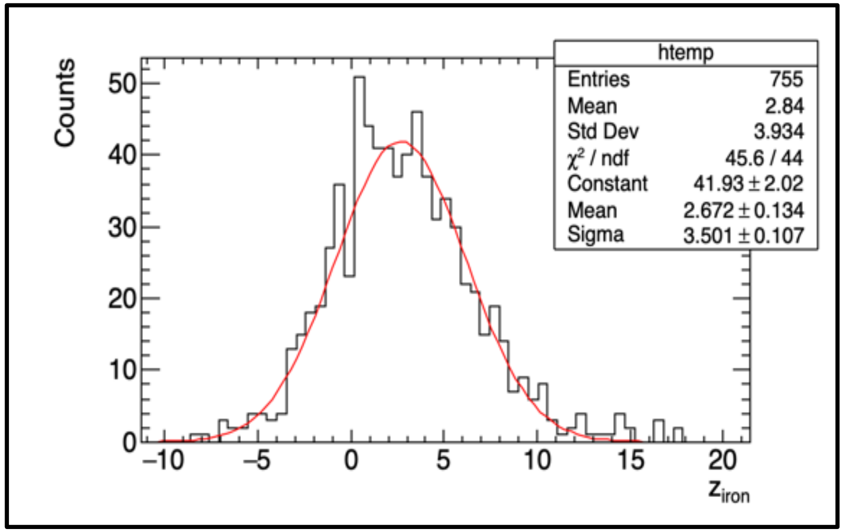


$\langle \mathbf{z} \rangle = \mathbf{8.30\ VEM},\ \sigma = \mathbf{4.2}$





$\langle \mathbf{z} \rangle = \mathbf{2.70\ VEM},\ \sigma = \mathbf{3.5}$



z-variable depends on primary particle type

Estimated muon signal at 1000 m (QGSJETII-04)-MC simulations

Signal fraction from muons from MC simulations

$$S^{MC}_{\mu,i,j}(\sec\theta) = g_{\mu,i}(\sec(\theta)) \times S^{MC}_{1000,i,j}(\sec(\theta))$$

Estimated muon signal at 1000 m

$S^{MC}(1000)$ from MC simulations with QGSJETII-04

j=event, i=primary type

Fraction of muons from MC simulations

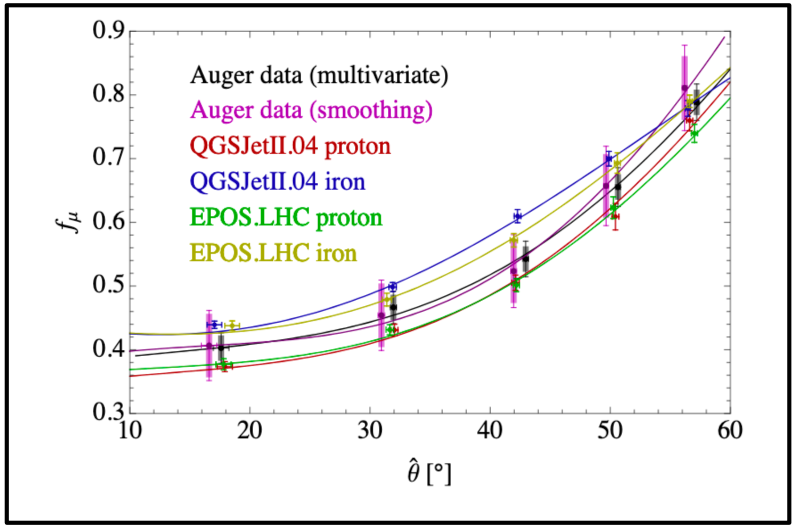
> The average fraction of the ground signal induced by muons has been calculated in many analyses. This fraction depends on the zenith angle and primary type, but only slightly on different hadronic interactions models.

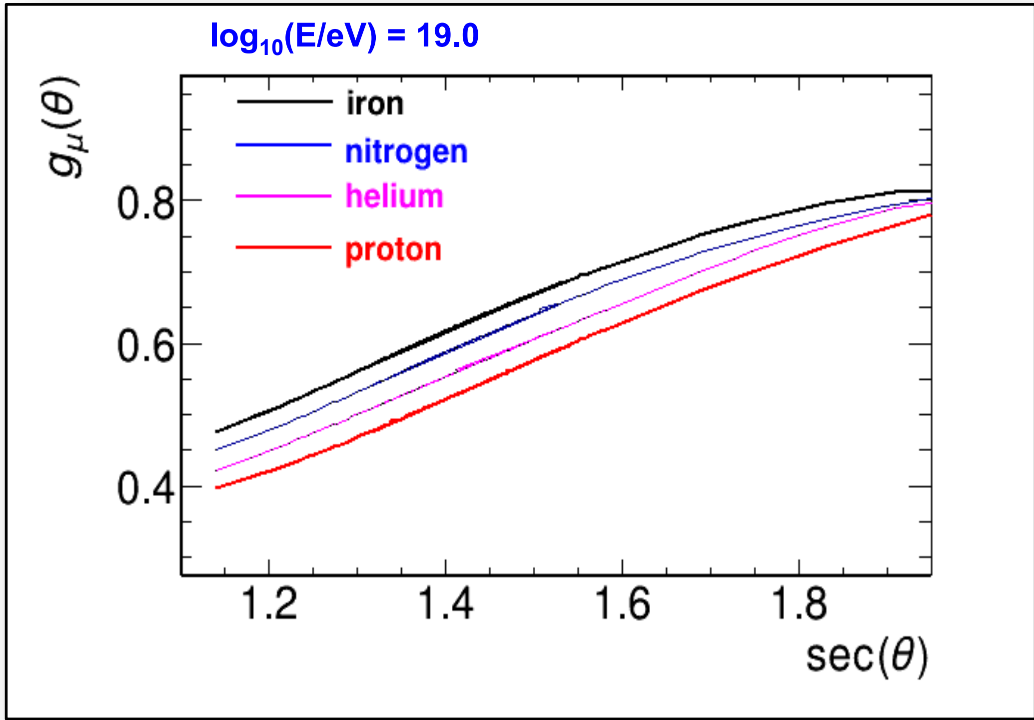
Muon FADC trace from q^th dense station in 1000m

$$\langle g_\mu \rangle := \frac{\langle S_\mu(1000)\rangle}{\langle S_{tot}(1000)\rangle} = \frac{\sum_q \sum T_\mu^{(q)}}{\sum_q \sum T^{(q)}}$$

Total FADC trace from q^th dense station in 1000m

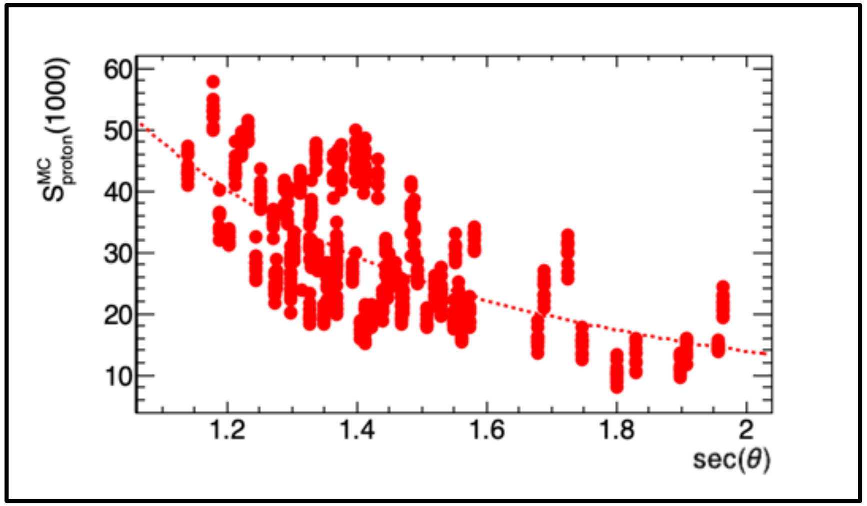
Balazs Kegl for the Pierre Auger Collab.,ICRC-2013 [astro-ph 1307.5059]



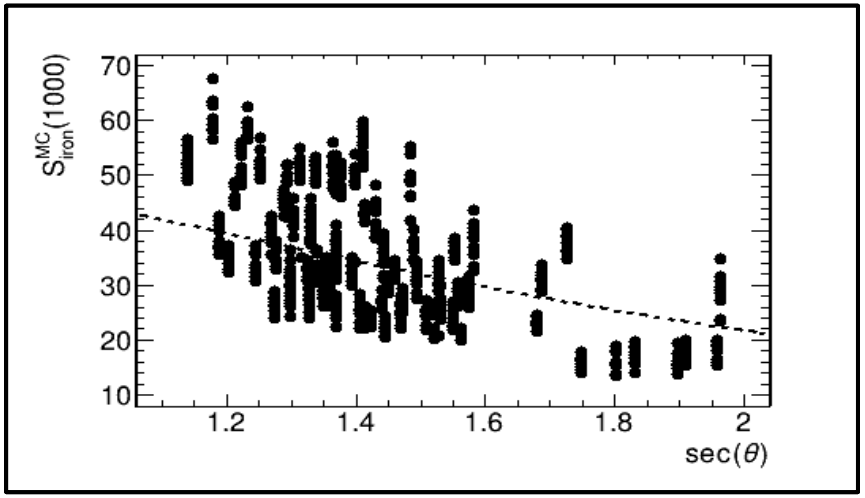


Distribution of $S^{MC}(1000)$ and S_{muon} as a function of zenith angle (QGSJETII-04)-MC

$\langle \mathbf{S^{MC}_{proton}} \rangle = \mathbf{28.78}$ VEM, $\sigma = \mathbf{10.2}$

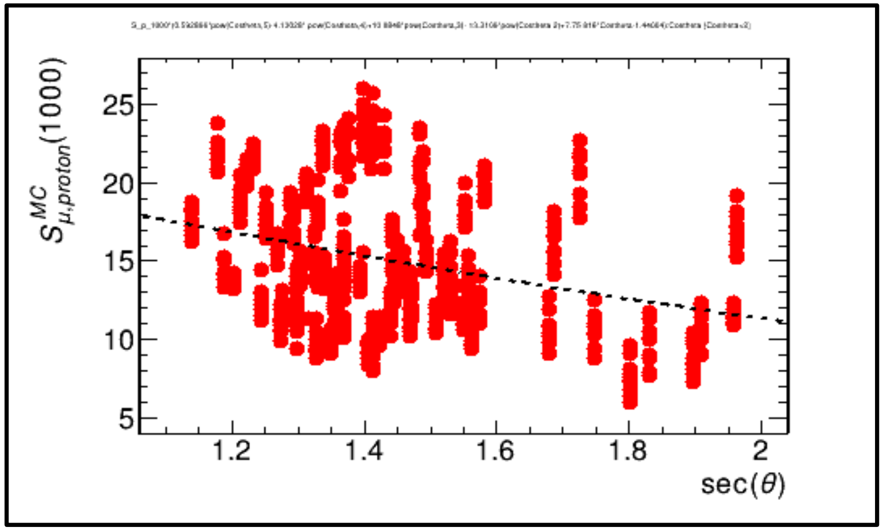


$\langle \mathbf{S^{MC}_{iron}} \rangle = \mathbf{34.36}$ VEM, $\sigma = \mathbf{11.3}$

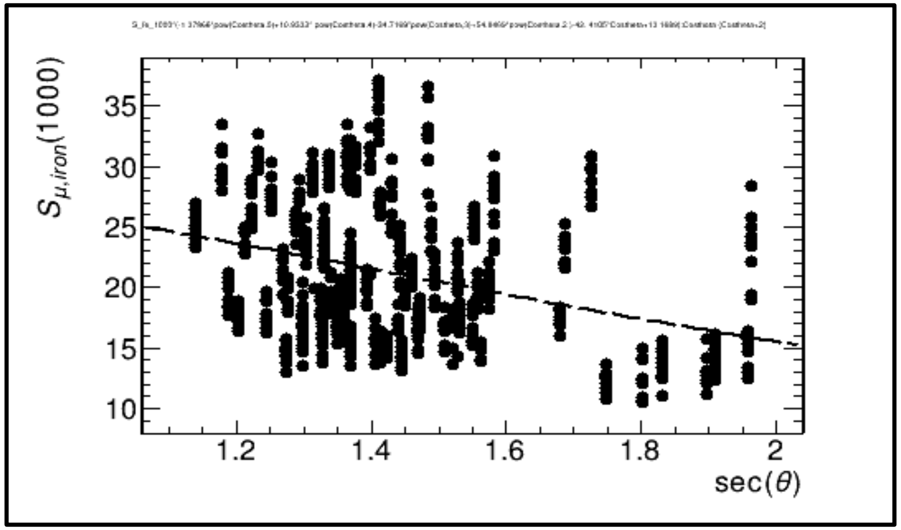


$\mathbf{g}_\mu(\theta)$ $\mathbf{g}_\mu(\theta)$

$\langle \mathbf{S^{MC}_{\mu,proton}} \rangle = \mathbf{15.05}$ VEM, $\sigma = \mathbf{4.3}$



$\langle \mathbf{S^{MC}_{\mu,iron}} \rangle = \mathbf{21.1}$ VEM, $\sigma = \mathbf{5.7}$



$R_{\mu,i,j}$ scaling factors from event-by-event calculations

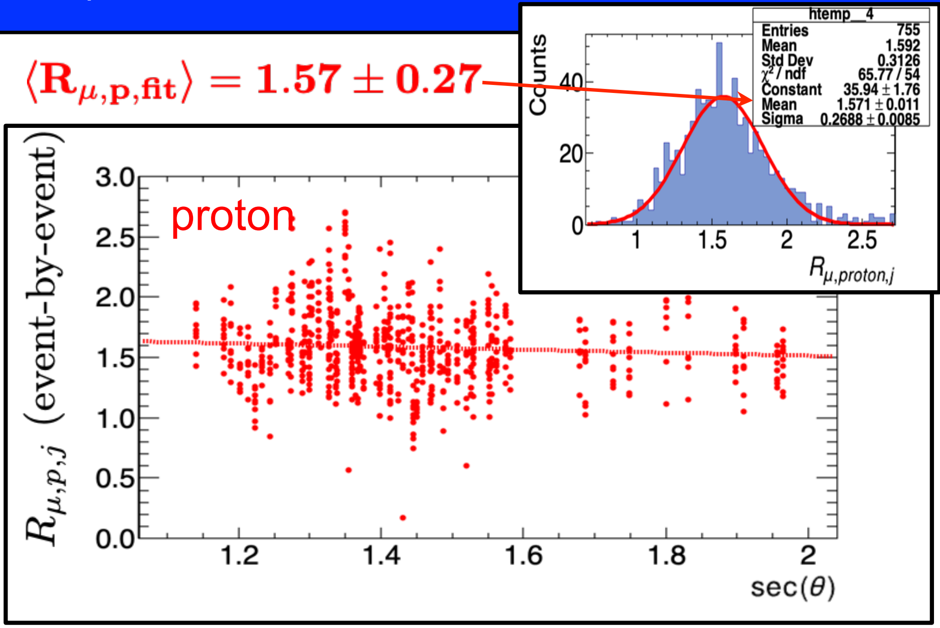
Having individual value of z_i from MC simulations, the corresponding SD signal at 1000 m, and using the parametrization of the muon fraction we can get the muon scaling factor for an individual hybrid event j

$$R_{\mu,i,j}(\sec(\theta)) = 1 + \frac{z_{i,j}(\sec(\theta))}{g_{\mu,i}(\sec(\theta)) \times S^{MC}_{1000,i,j}(\sec(\theta))} \text{ for } R_E = 1$$

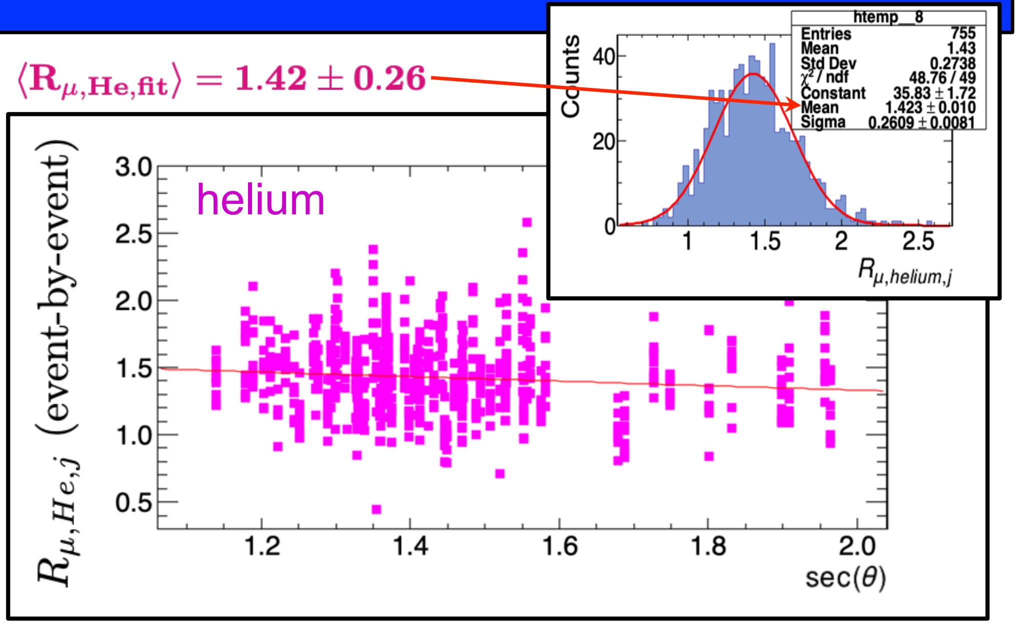
j=event, i=primary type

R_{μ}: event-by-event analysis (EPOS-LHC as MOCK DATA SET)

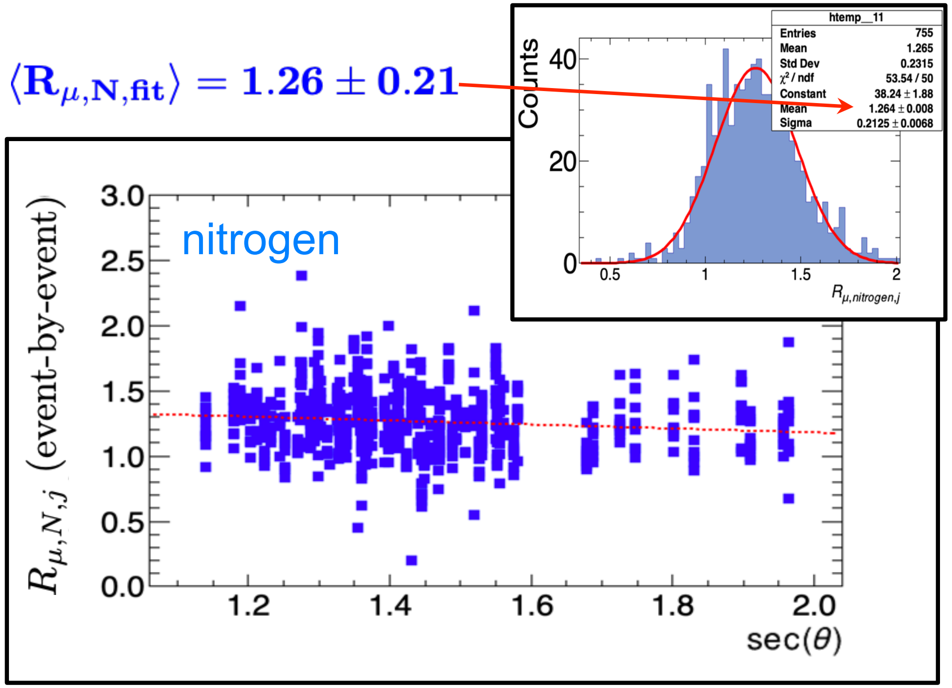
$\langle \mathbf{R_{\mu,p,fit}} \rangle = \mathbf{1.57 \pm 0.27}$



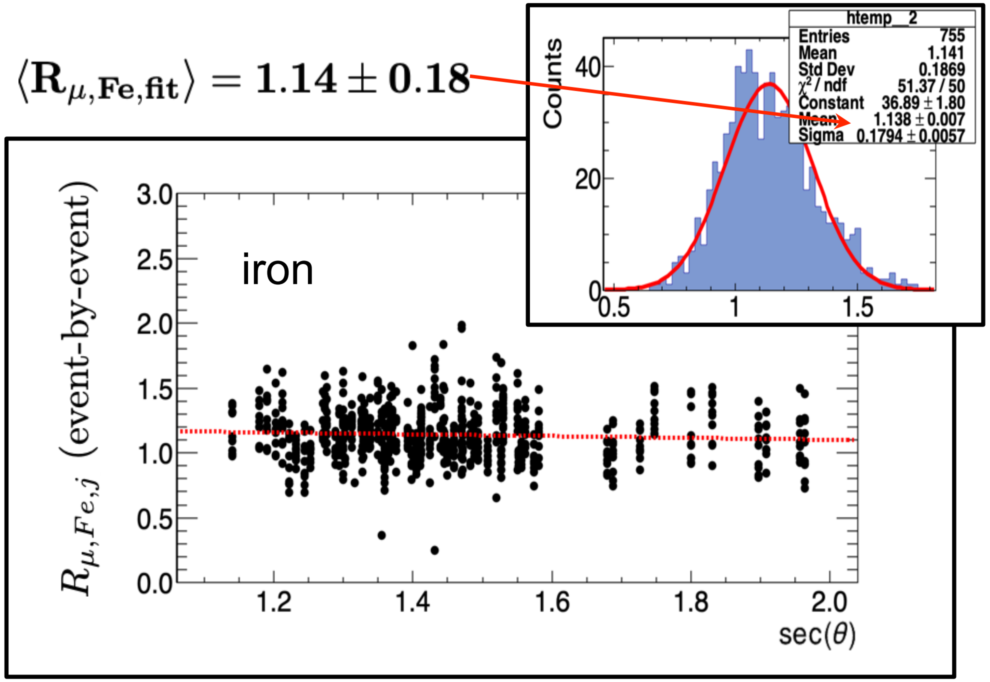
$\langle \mathbf{R_{\mu,He,fit}} \rangle = \mathbf{1.42 \pm 0.26}$



$\langle \mathbf{R_{\mu,N,fit}} \rangle = \mathbf{1.26 \pm 0.21}$



$\langle \mathbf{R_{\mu,Fe,fit}} \rangle = \mathbf{1.14 \pm 0.18}$



Muon scaling parameter R_μ (EPOS-LHC as MOCK-DATA)

Table 1: The mean value of the muon rescaling parameters $R_{\mu,i}$ calculated from Eq. (4) and its standard deviation $\sigma_{\langle R_{\mu,i}\rangle}$ for different i primaries. Also, the corresponding mean values of the total muon SD signal $S^{MC}_{1000,\mu,i}$ from QGSJetII-0.4 model, reconstructed muon SD signal at 1000 m expected in the MOCK-DATA set and the ratio $k \equiv (\langle R_{\mu,i}\rangle \times \langle S^{MC}_{1000,\mu}\rangle - S^{MC-True}_{\mu})/S^{MC-True}_{\mu}$ are listed.

primary type i	$\langle R_{\mu,i}\rangle$	$\sigma_{\langle R_{\mu,i}\rangle}$	$\langle S^{MC}_{1000,\mu,i}\rangle$ [VEM]	$\langle R_{\mu,i}\rangle \times \langle S^{MC}_{1000,\mu,i}\rangle$ [VEM]	k [%]
p	1.57 ± 0.01	0.27	15.05	23.63	+2.3
He	1.42 ± 0.01	0.26	16.82	23.88	+3.4
N	1.26 ± 0.01	0.21	18.96	23.89	+3.5
Fe	1.14 ± 0.01	0.18	21.08	24.00	+3.9

We need to rescale the QGSJETII-04 muon signal about **factor ~1.6 (proton) and 1.1 (iron)** in order to get the muon signal for iron in EPOS-LHC. This is consistent with expectation: T. Pierog, Eur. Phys. J. Web Conf. 52, 03001 (2013).

The method can reproduce an average muon signal calculated for iron with EPOS-LHC within 2 - 4%

Results: the beta exponent

From Heitler-Matthews model Astropart. Phys. 22, 387 (2005):

$$N_\mu \simeq A(\frac{E/A}{\epsilon_c^\pi})^\beta = N_\mu^p A^{1-\beta} \qquad \ln\langle N_\mu\rangle = \ln\langle N_{\mu,p}\rangle + (1-\beta)\ln A$$

where ϵ_d^π is the critical energy at which pions decays into muons and $\beta \simeq 0.9$

- In L. Calzon et all., Phys. Lett. B784, 68–76 (2018) was reported β≈0.927 (EPOS-LHC) and β≈0.925 (QGSJet II-04).

- Detailed MC simulations of the β show dependence on hadronic-interaction properties, like the multiplicity, the charge ratio and the baryon anti-baryon pair production R. Ulrich et al., PRD 83, 054026 (2011).

Assuming that $\langle N_\mu\rangle \propto \langle S_\mu\rangle$, we can replace ln $\langle N_\mu\rangle$ by ln $\langle S_\mu\rangle$:

> Beta exponent (MC) from TD simulations: cross-check:

from Table shown on slide 11

$$\beta_p = 1 - \frac{\ln\langle S_{\mu,\mathbf{Fe}}^{MC}\rangle - \ln\langle S_{\mu,p}{}^{MC}\rangle}{\ln 56} = 1 - \frac{\ln(21.1) - \ln(15.1)}{\ln 56} = \mathbf{0.92} \quad \text{also} \quad \beta_{He} \simeq \beta_N \simeq 0.92$$

> Beta exponent using rescaling muon factor:

$$S_{\mu,i}^{\rm MOCK-DATA} \equiv \langle R_{\mu,i}\rangle \times \langle S_{1000,\mu,i}^{\rm MC}\rangle$$

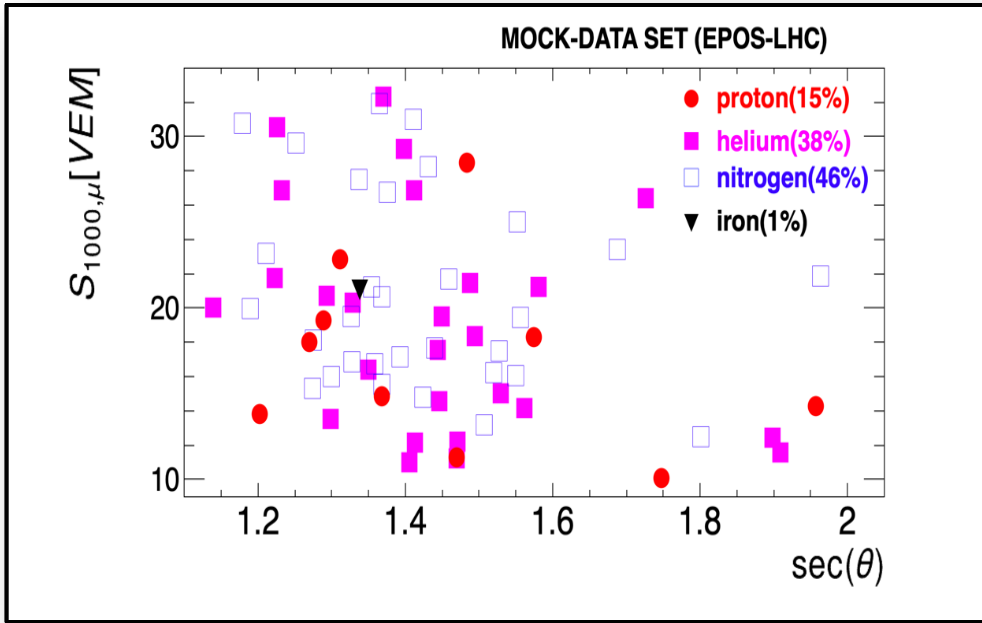
$$\beta_i = 1 - \frac{\ln\langle S_{\mu,Fe}^{\rm MOCK-DATA}\rangle - \ln\langle S_{\mu,i}^{\rm MOCK-DATA}\rangle}{\ln 56 - \ln A_i} = 1 - \frac{\ln(R_{\mu,Fe}\langle S_{\mu,Fe}^{\rm MC}\rangle) - \ln(R_{\mu,i}\langle S_{\mu,i}^{\rm MC}\rangle)}{\ln 56 - \ln A_i}$$

Method to measure the beta exponent

1) Generation of a new MOCK-DATA set,
in such a way that considered fraction of events follow AUGER predictions for EPOS-LHC :

$$\mathbf{f_p \simeq 15\%, f_{He} \simeq 38\%, f_N \simeq 46\%, f_{Fe} \simeq 1\%}$$

J. Bellido et al., PoS(ICRC2017)506 (2017).

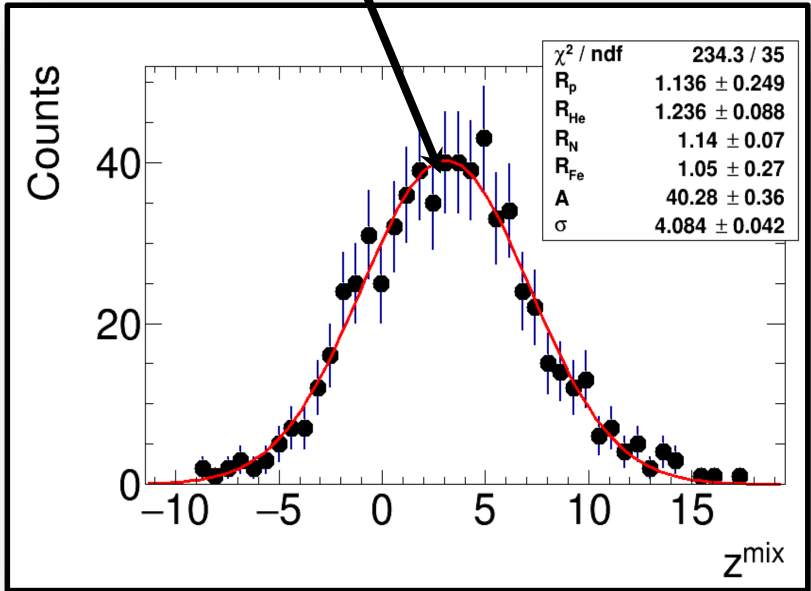


2) z-variable for mixed composition

$$z_j^{\rm mix} \equiv S_{1000,j}^{\rm MOCK-DATA} - \sum_i f_i S_{\mu,i,j}^{\rm MC}$$

3) The Gaussian fit to z^{mix}-histogram

$$P(A,\sigma,\mathbf{R}_\mu^{\rm fit}) = A\exp(-(z^{\rm mix} - \langle z^{\rm mix}\rangle)^2/2\sigma^2)$$
$$\text{where } \langle z^{\rm mix}\rangle = \sum f_i \times \langle S_{\mu,i}^{\rm MC}\rangle (R_{\mu,i}^{\rm fit} - 1)$$



4) The solution should fulfill the following cuts:

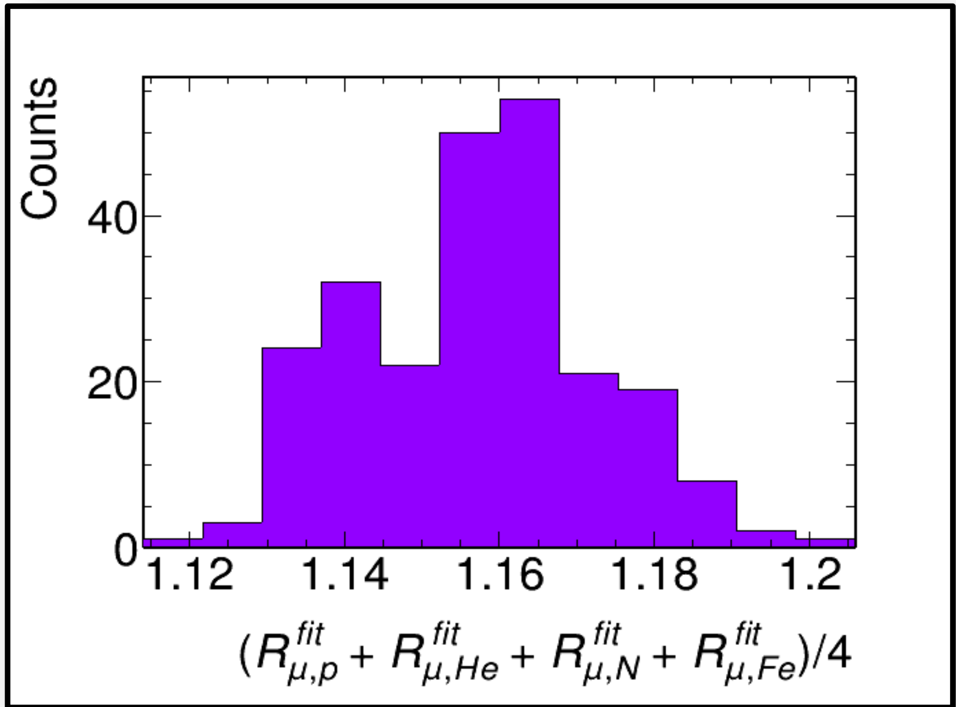
- the average muon numbers for lighter elements should be smaller than for heavier elements:

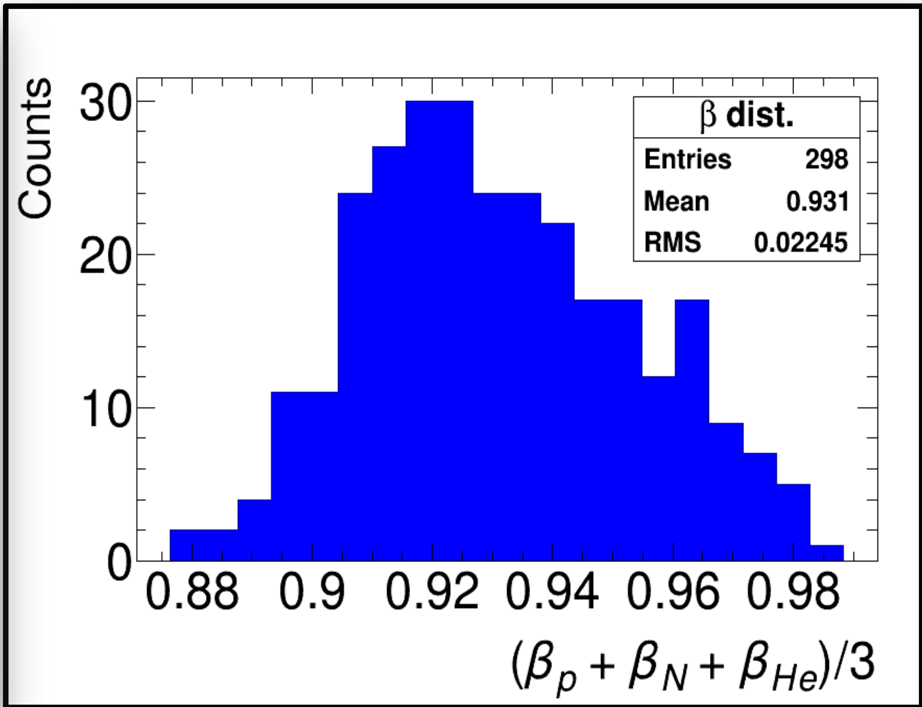
$$R_{\mu,\rm p}\langle S_{\mu,\rm p}^{\rm MC}\rangle < R_{\mu,\rm He}\langle S_{\mu,\rm He}^{MC}\rangle < R_{\mu,\rm N}\langle S_{\mu,\rm N}^{\rm MC}\rangle < R_{\mu,\rm Fe}\langle S_{\mu,\rm Fe}^{\rm MC}\rangle$$

- to account for different cosmic-ray compositions derived using EPOS and QGSJet models:

$$R_{\mu,\rm He}^{\rm fit} > R_{\mu,i}^{\rm fit} \text{ where } i \in \{\rm p, N, Fe\}$$

Method to measure the beta exponent: results





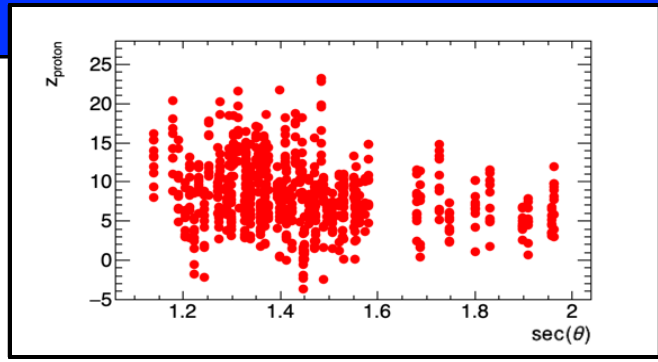
We can recover:

- the ratio in the muon signal between EPOS-LHC and QGSJetII-0.4, on average within -5%,.

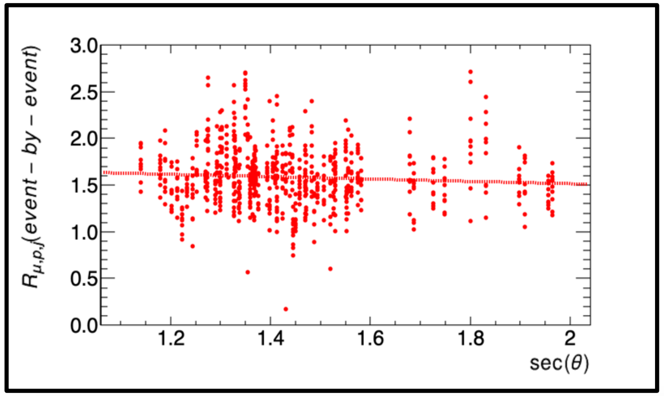
-the parameter β = 0.92 for the studied system, which is a consequence of the good recovery (less than 6% on average) of the muon signal for each primary.

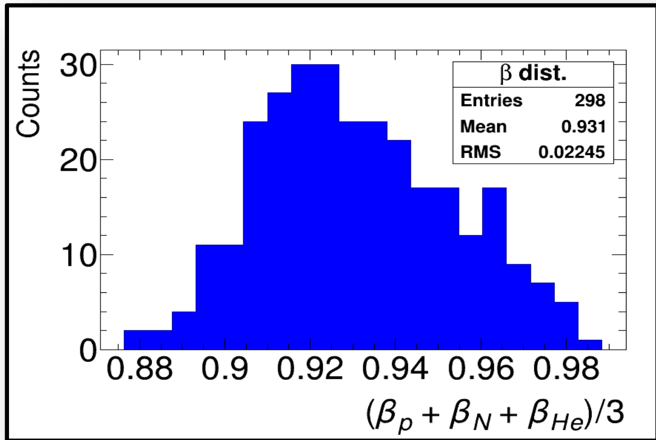
Summary

$$z_j \equiv S_{1000,j}^{\mathrm{MOCK-DATA}} - S_{1000,j}^{\mathrm{MC}}$$



$$R_{\mu,i,j}(\theta) = 1 + \frac{z_{i,j}(\sec(\theta))}{g_{\mu,i}(\theta) \times S_{1000,i,j}^{MC}(\sec(\theta))} \quad \text{for } R_E{=}1$$





> We present a new method to derive muon rescaling factors by analyzing reconstructions of simulated showers.

> The z-variable used is connected to the muon signal, and is roughly independent of the zenith angle, but depends on the mass of primary cosmic ray.

- The performance of the method is tested by using Monte Carlo shower simulations for the hybrid detector of the Pierre Auger Observatory.

- Having an individual z-value from each simulated hybrid event, the corresponding signal at 1000 m, S^{MC}, and using a parametrization of the muon fraction, g_{mu} in simulated showers, we can calculate the multiplicative rescaling parameters of the muon signals in the ground detector even for an individual event, and study its dependence as a function of zenith angle and the mass of primary cosmic ray.

> This gives a possibility not only to test/calibrate the hadronic interaction models, but also to derive the beta exponent, describing increase of the number of muons as a function of primary energy and cosmic-ray mass.

ICRC 2021- #412

Acknowledgments: The authors are very grateful to the Pierre Auger Collaboration for providing the simulations for this contribution.
We would like also to thank Kevin Almeida Cheminant for various cross-checks of the presented analysis