On the muon scale of air showers and its application to the AGASA data

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Motivation and objective

Evidence of a muon deficit in air shower simulations [1, 2, 3, 4].

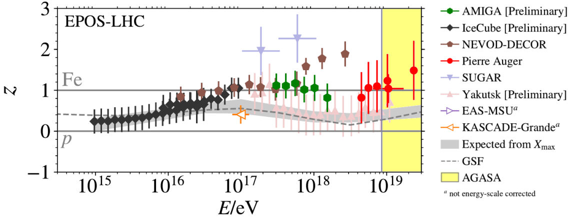

Fig. 1: Image adapted from Ref. [2].

Muon scale

num. of muons from data

from p det. sim.

$$z := \frac{\ln N_{\mu,\,\mathrm{data}}^{\mathrm{det}} - \ln N_{\mu,\,\mathrm{p}}^{\mathrm{det}}}{\ln N_{\mu,\,\mathrm{Fe}}^{\mathrm{det}} - \ln N_{\mu,\,\mathrm{p}}^{\mathrm{det}}}$$

from Fe det. sim.

from p det. sim.

Objectives

1. Define two estimators of z, study their properties & syst.
2. Compute their values from AGASA data.

$z_{\ln\langle\cdot\rangle}$ and $z_{\langle\ln\cdot\rangle}$

We introduce the average over an energy bin in two ways:

$$z_{\ln\langle\cdot\rangle} = \frac{\ln\langle N^{\rm det}_{\mu,\,\rm data}\rangle - \ln\langle N^{\rm det}_{\mu,\,\rm p}\rangle}{\ln\langle N^{\rm det}_{\mu,\,\rm Fe}\rangle - \ln\langle N^{\rm det}_{\mu,\,\rm p}\rangle} \qquad (1) \qquad\qquad z_{\langle\ln\cdot\rangle} = \frac{\langle\ln N^{\rm det}_{\mu,\,\rm data}\rangle - \langle\ln N^{\rm det}_{\mu,\,\rm p}\rangle}{\langle\ln N^{\rm det}_{\mu,\,\rm Fe}\rangle - \langle\ln N^{\rm det}_{\mu,\,\rm p}\rangle} \qquad (2)$$

$$\neq$$

$$\downarrow$$

$$\langle\ln N_\mu\rangle \approx \ln\langle N_\mu\rangle - \tfrac{1}{2}\left(\mathrm{RSD}_{\rm tot}[N_\mu]\right)^2 , \tag{3}$$

where $\mathrm{RSD}_{\underset{\substack{\uparrow \\ \text{all sources}}}{\rm tot}}[N_\mu] = \sigma_{\rm tot}(N_\mu)/\left\langle N_\mu\right\rangle$ is the total Relative Std. Dev. of N_μ [5].

Experiments with different resolution: same $\ln\left\langle N^{\rm det}_{\mu,\rm data}\right\rangle \rightarrow$ different $\left\langle\ln N^{\rm det}_{\mu,\rm data}\right\rangle$

→ $z_{\ln\langle\cdot\rangle}$ is better for comparing different experiments

Systematics in the muon scale estimators

Sources of systematics	Syst. in $z_{\ln\langle\cdot\rangle}$	Syst. in $z_{\langle\ln\cdot\rangle}$
Composition bias in data $\langle N_{\mu,\,\mathrm{data}}^{\mathrm{det}}\rangle$	✓	✓
Mismodeled detector effects in the mean $\langle N_{\mu,\{\mathrm{p,Fe}\}}^{\mathrm{det}}\rangle$	✓	✓
Mismodeled detector resolution		✓

Systematics in the muon scale estimators

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Mismodeled detector effects in the mean $\langle N_{\mu,\{\mathrm{p},\mathrm{Fe}\}}^{\mathrm{det}}\rangle$	✓	✓
Mismodeled detector resolution		✓

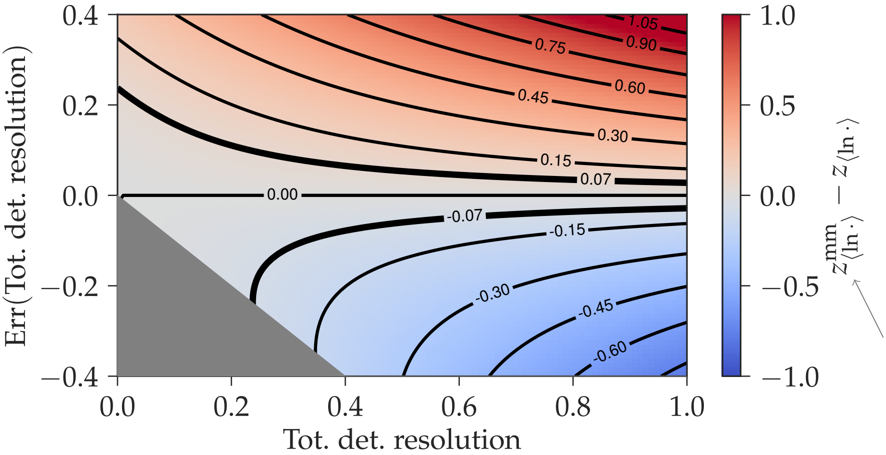
Syst. in $z_{\langle \ln \cdot \rangle}$ from mismodeled detector resolution

If we assume

- No mismodeled detector effects in the mean $\langle N_{\mu,\{\mathrm{p,Fe}\}}^{\mathrm{det}} \rangle$ (already in syst.).
- $\langle \ln N_\mu \rangle \approx \ln\langle N_\mu \rangle - \frac{1}{2}\left(\mathrm{RSD}_{\mathrm{tot}}[N_\mu]\right)^2$ (Eq. (3)).
- Heitler-Matthews model (see Eq. (4))

Then...

Syst. in $z_{\langle \ln \cdot \rangle}$ from mismodeled detector resolution



$z_{\langle \ln \cdot \rangle}$ with
mis-modeled
det. res.

Fig. 2: Syst. err. in $z_{\langle \ln \cdot \rangle}$ as a function of the true det. res. (x-axis) and of the difference between the mismodeled and true det. res. (y-axis).

Syst. in $z_{\langle \ln \cdot \rangle}$ from mismodeled detector resolution

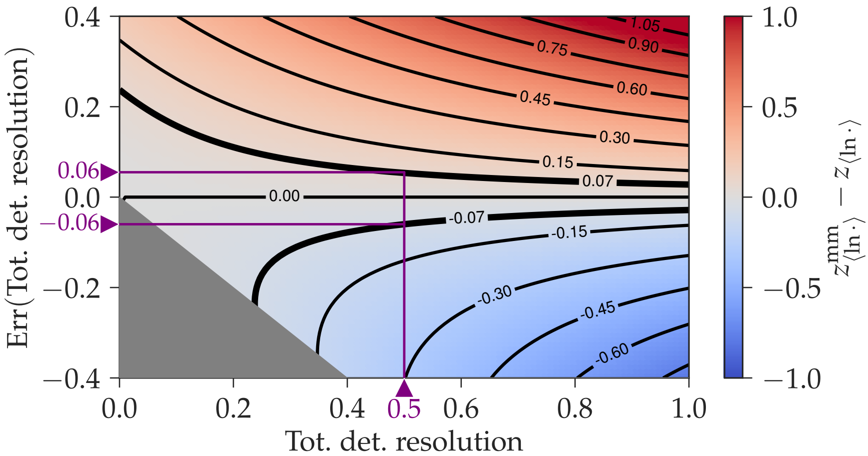


Fig. 3: E.g.: The total det. resolution of AGASA is $\sim 50\,\%$ in the best case. Syst. err of ± 0.07 in $z_{\langle \ln \cdot \rangle}$ is attained when the resolution is mismodeled in $\pm 6\,\%$.

Muon deficit scale

To compute a muon deficit scale, we need a reference.

Heitler-Matthews model [6]

mass

primary energy

$$\ln N_\mu = (1-\beta)\ln A + \beta \ln(E/\xi_c), \tag{4}$$

power-law index $\beta \approx 0.9$

critical energy constant

In the Heitler-Matthews model: $z^{\mathrm{HM}}_{\langle \ln \cdot \rangle\ \mathrm{mass}} = \dfrac{\langle \ln A \rangle}{\ln 56}$.

using mass composition model

We therefore compute:

$$\Delta z_{\ln\langle\cdot\rangle} = z_{\ln\langle\cdot\rangle} - z^{\mathrm{HM}}_{\langle \ln \cdot \rangle\ \mathrm{mass}} \tag{5}$$

$$\Delta z_{\langle \ln \cdot \rangle} = z_{\langle \ln \cdot \rangle} - z^{\mathrm{HM}}_{\langle \ln \cdot \rangle\ \mathrm{mass}} \tag{6}$$

Systematics in the muon deficit estimators

Sources of systematics	Syst. in $\Delta z_{\ln\langle\cdot\rangle}$	Syst. in $\Delta z_{\langle\ln\cdot\rangle}$
Propagated from $z_{\ln\langle\cdot\rangle}$ or $z_{\langle\ln\cdot\rangle}$	✓	✓
Syst. in the composition model in $z^{\mathrm{HM}}_{\langle\ln\cdot\rangle\,\mathrm{mass}}$	✓	✓
Deviations from the H.M. model in $z^{\mathrm{HM}}_{\langle\ln\cdot\rangle\,\mathrm{mass}}$	✓	✓
Bias from shower-to-shower fluctuations	✓	

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Deviations from the H.M. model in $z^{\mathrm{HM}}_{\langle\ln\cdot\rangle\,\mathrm{mass}}$	✓	✓
Bias from shower-to-shower fluctuations	✓	

Bias in $\Delta z_{\ln\langle\cdot\rangle}$ from sh-sh fluctuations in $z^{\mathrm{HM}}_{\langle\ln\cdot\rangle\,\mathrm{mass}}$

- If $\Delta z_{\ln\langle\cdot\rangle}$ were computed taking $z_{\ln\langle\cdot\rangle\,\mathrm{mass}}$ as reference, there would be no bias.
- However, we take as a reference $z^{\mathrm{HM}}_{\langle\ln\cdot\rangle\,\mathrm{mass}}$, the predicted value of $z_{\langle\ln\cdot\rangle\,\mathrm{mass}}$.

det. sim. using mass composition model

$$z_{\langle\ln\cdot\rangle\,\mathrm{mass}} = \frac{\langle\ln N^{\mathrm{det}}_{\mu,\,\mathrm{mass}}\rangle - \langle\ln N^{\mathrm{det}}_{\mu,\,\mathrm{p}}\rangle}{\langle\ln N^{\mathrm{det}}_{\mu,\,\mathrm{Fe}}\rangle - \langle\ln N^{\mathrm{det}}_{\mu,\,\mathrm{p}}\rangle} \tag{7}$$

$$\approx \frac{\ln\langle N^{\mathrm{det}}_{\mu,\,\mathrm{mass}}\rangle - \ln\langle N^{\mathrm{det}}_{\mu,\,\mathrm{p}}\rangle - \frac{1}{2}\left[(\mathrm{RSD}_{\text{sh-sh}}[N_{\mu,\,\mathrm{mass}}])^2 - (\mathrm{RSD}_{\text{sh-sh}}[N_{\mu,\,\mathrm{p}}])^2\right]}{\ln\langle N^{\mathrm{det}}_{\mu,\,\mathrm{Fe}}\rangle - \ln\langle N^{\mathrm{det}}_{\mu,\,\mathrm{p}}\rangle - \frac{1}{2}\left[(\mathrm{RSD}_{\text{sh-sh}}[N_{\mu,\,\mathrm{Fe}}])^2 - (\mathrm{RSD}_{\text{sh-sh}}[N_{\mu,\,\mathrm{p}}])^2\right]} \tag{8}$$

$z_{\ln\langle\cdot\rangle\,\mathrm{mass}}$

sh-sh flucutations

→ $z_{\langle\ln\cdot\rangle\,\mathrm{mass}}$ depends on sh-sh fluctuations, while $z_{\ln\langle\cdot\rangle\,\mathrm{mass}}$ does not.

Worst case scenario: $\sim 50\,\%$ p $+ 50\,\%$ Fe → $|z_{\ln\langle\cdot\rangle\,\mathrm{mass}} - z_{\langle\ln\cdot\rangle\,\mathrm{mass}}| \leq 0.07$ [5].
Furthermore, this bias in $\Delta z_{\ln\langle\cdot\rangle}$ can be corrected.

Overall systematics comparison

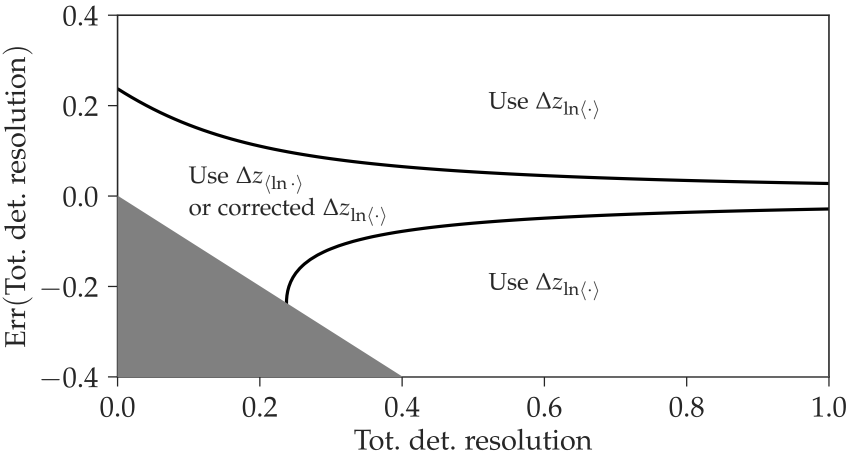


Fig. 4: In typical scenarios, it is better to use $\Delta z_{\ln\langle\cdot\rangle}$.

The muon scale from AGASA data

Akeno Giant
Air Shower Array

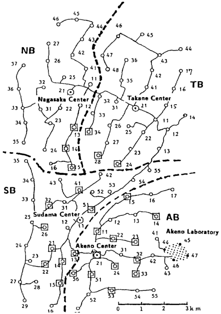


Fig. 5: Image extracted from Ref. [7].

Data

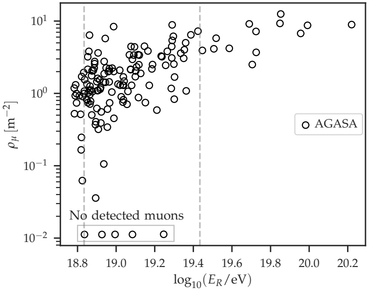


Fig. 6: Data extracted from Ref. [8]. Energy is rescaled to reference scale [9, 1].

Simulations

p
He
N
Fe

+

EPOS-LHC
QGSJetII-04
Sibyll2.3c

Sim. library described in Ref. [1].
Det. effects are accounted for analytically.

Results

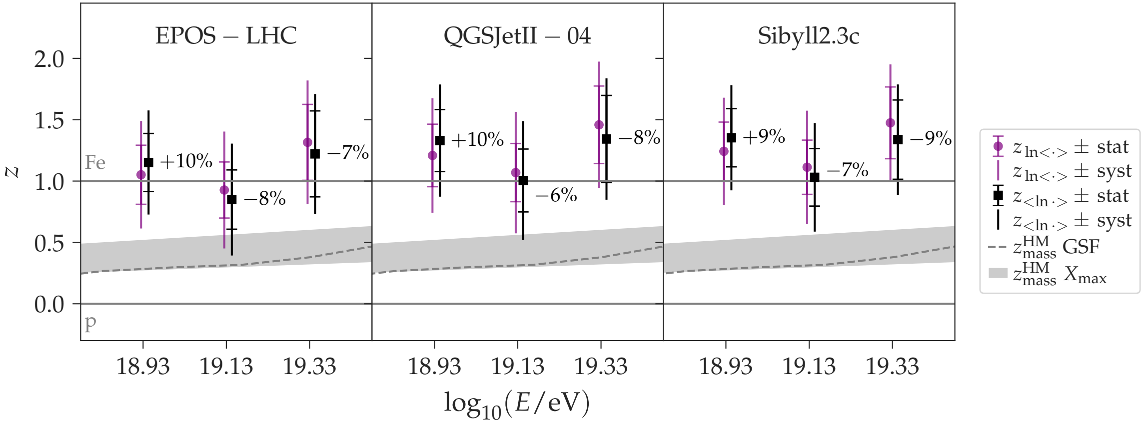
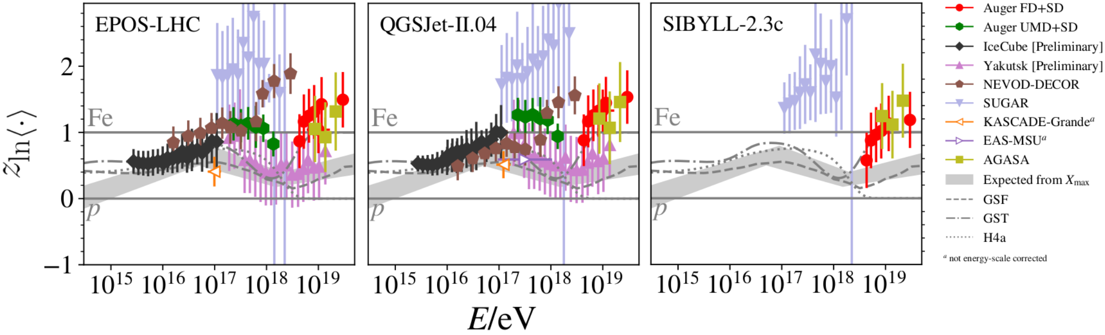


Fig. 7: $z_{\ln\langle\cdot\rangle} = \frac{\ln\langle\rho^{\mathrm{det}}_{\mu,\mathrm{data}}\rangle - \ln\langle\rho_{\mu,\mathrm{p}}\rangle}{\ln\langle\rho_{\mu,\mathrm{Fe}}\rangle - \ln\langle\rho_{\mu,\mathrm{p}}\rangle}$ vs $z_{\langle\ln\cdot\rangle} \approx \frac{\langle\ln\rho^{\mathrm{det}}_{\mu,\mathrm{data}}\rangle - \ln\langle\rho_{\mu,\mathrm{p}}\rangle + \frac{1}{2}\left[(\mathrm{RSD}_{\text{sh-sh}}[\rho_{\mu,\,\mathrm{p}}])^2 + (\mathrm{RSD}_{\text{not sh-sh}}[\rho_{\mu}])^2\right]}{\ln\langle\rho_{\mu,\mathrm{Fe}}\rangle - \ln\langle\rho_{\mu,\mathrm{p}}\rangle + \frac{1}{2}\left[(\mathrm{RSD}_{\text{sh-sh}}[\rho_{\mu,\,\mathrm{p}}])^2 - (\mathrm{RSD}_{\text{sh-sh}}[\rho_{\mu,\,\mathrm{Fe}}])^2\right]}$

Results

Fig. 8: Image adaped from Ref. [4].



There is an agreement with Pierre Auger and Yakutsk array data.

AGASA data support a muon deficit in air shower simulations.

Summary

Muon scale: $z_{\ln\langle\cdot\rangle}$ vs $z_{\langle\ln\cdot\rangle}$

- $z_{\langle\ln\cdot\rangle}$ suffers from systematics if the detector resolution is mismodeled.
- We provided a way to estimate this systematic for any experiment.

$z_{\ln\langle\cdot\rangle}$ is always better for comparing results from different experiments.

Muon deficit scale: $\Delta z_{\ln\langle\cdot\rangle}$ vs $\Delta z_{\langle\ln\cdot\rangle}$

- $\Delta z_{\ln\langle\cdot\rangle}$ has a bias from shower-to-shower fluctuations in $z^{\rm HM}_{\ln\langle\cdot\rangle\,\rm mass} = \langle\ln A\rangle/\ln 56$.
- The bias in $\Delta z_{\ln\langle\cdot\rangle}$ is typically smaller than the syst. in $\Delta z_{\langle\ln\cdot\rangle}$ + correctible.

$\Delta z_{\ln\langle\cdot\rangle}$ is generally better for estimating the deficit.

***z* from AGASA data**

- They are in agreement with Pierre Auger and Yakutsk Array data.

AGASA data constitute further evidence of a muon deficit.

References I

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