A Complete Model of the Signal in Surface Detector Arrays and its Application for the Reconstruction of Mass-sensitive Observables

Max Stadelmaier
Markus Roth, David Schmidt, Darko Veberič

Karlsruher Institut für Technologie
UNIVERSIDAD NACIONAL DE SAN MARTÍN
HIRSAP
Double Doctoral degree in Astrophysics DDAp

max.stadelmaier@kit.edu

Goal

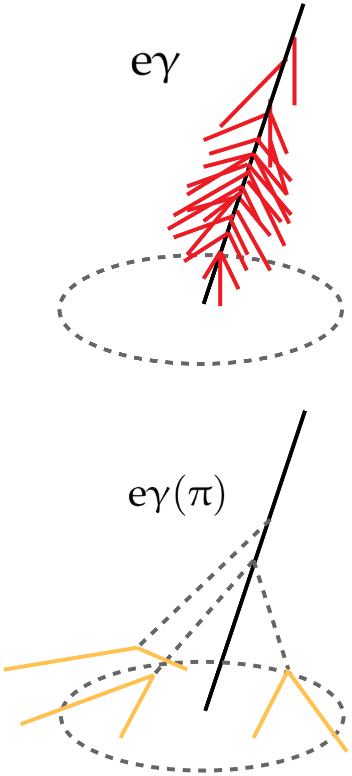
Reconstruct X_{max} and R_μ using **only** data from surface detector arrays.

How?

Find the mapping

$$X_{max}, R_\mu \mapsto S(X_{max}, R_\mu)$$

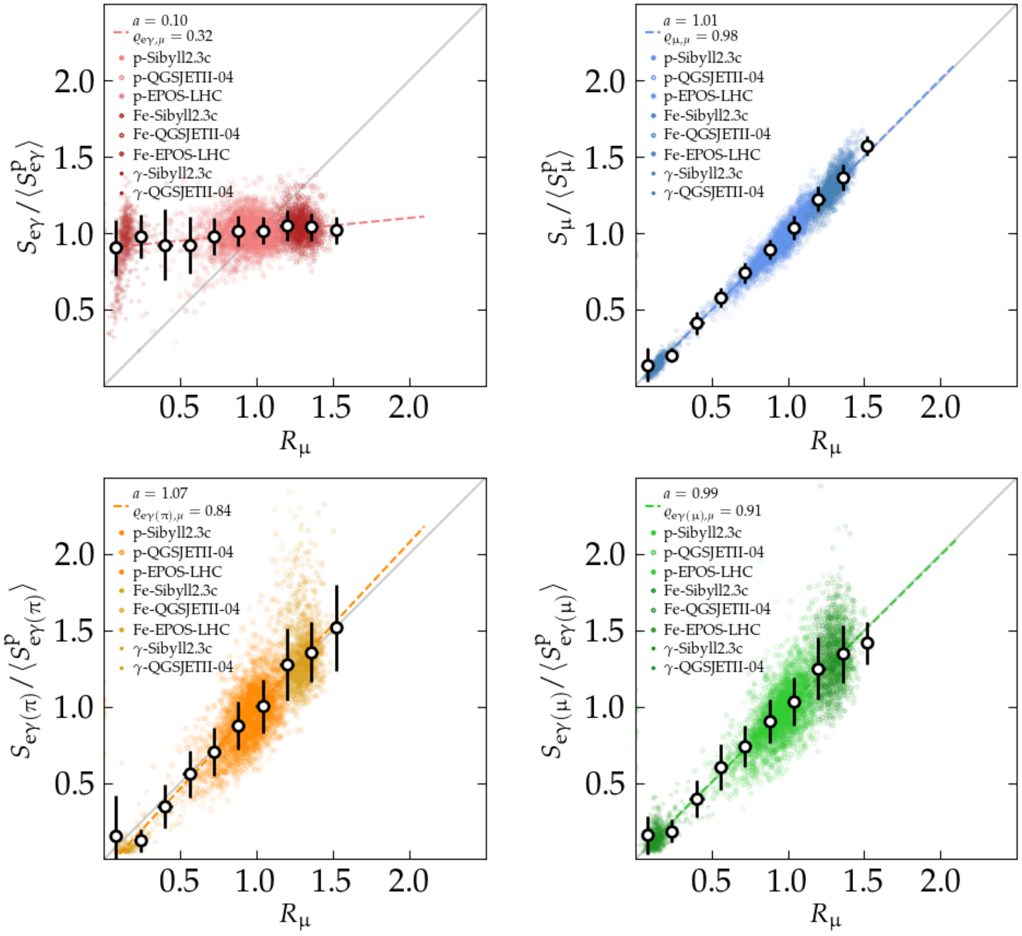
The four components



$$N_i = R_i \langle N_i^{\mathrm{p}} \rangle$$

$$N_\mu = R_\mu \langle N_\mu^{\mathrm{p}} \rangle$$

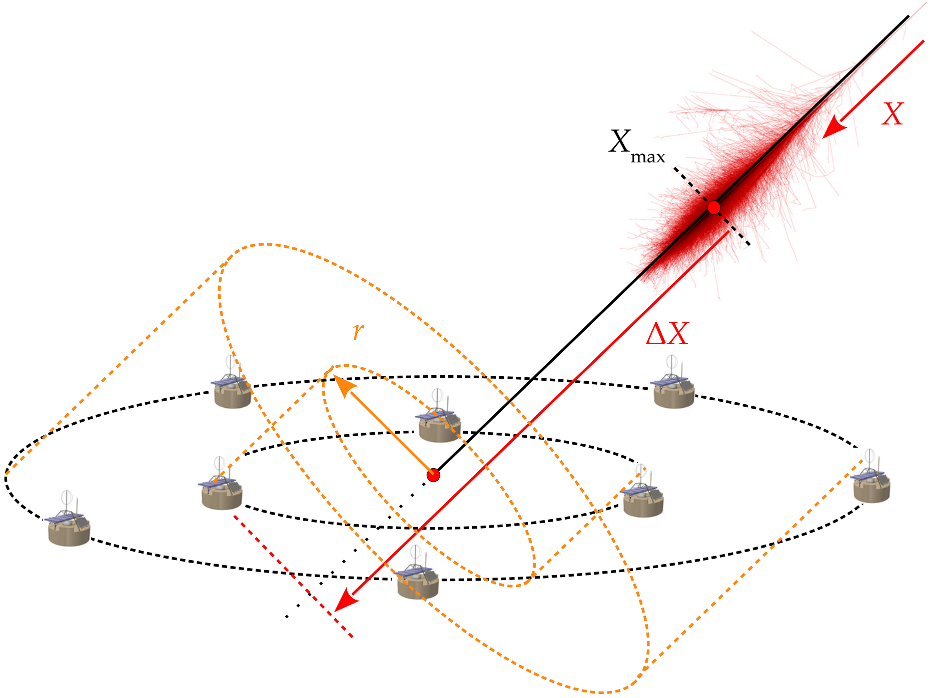
The four components



$$N_i = R_i \langle N_i^{p} \rangle$$

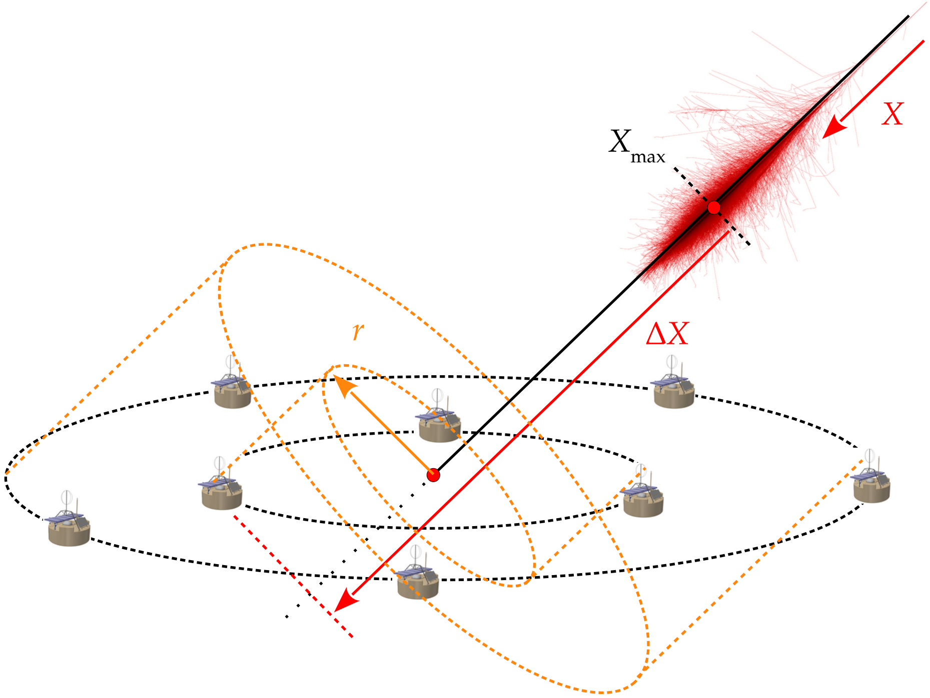
$$N_\mu = R_\mu \langle N_\mu^{p} \rangle$$

The lateral and longitudinal profile



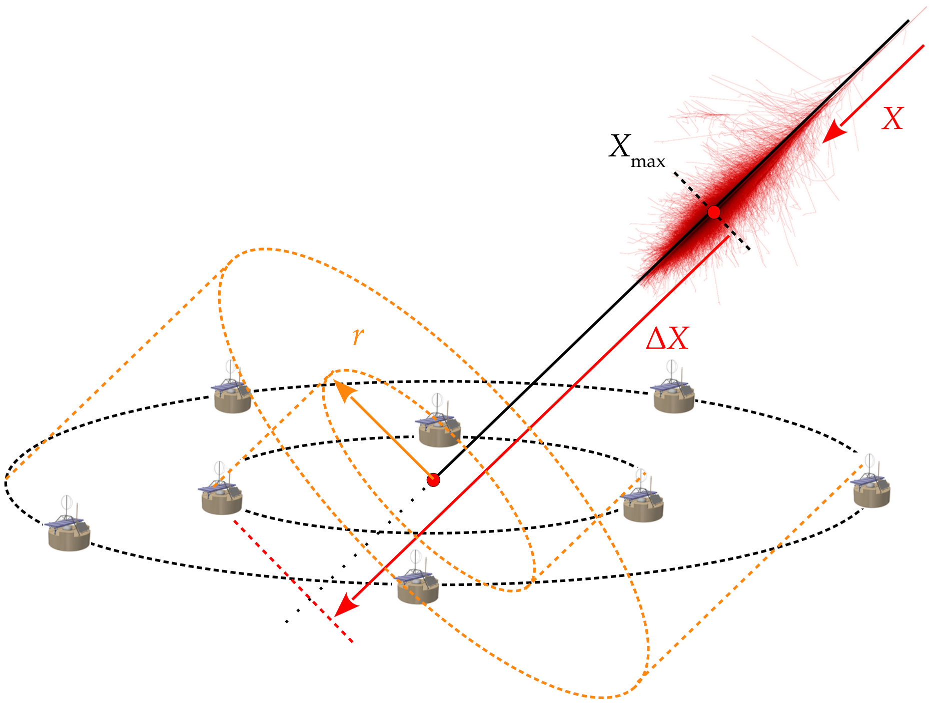
$$N(X) = N_{max} \left(\frac{X - X_1}{X_{max} - X_1} \right)^{\frac{X_{max} - X_1}{\lambda}} e^{-\frac{X - X_{max}}{\lambda}}$$

The lateral and longitudinal profile



$$\varrho(r,\Delta X) = \varrho(r)_{\mathrm{ref}} \left(\frac{\Delta X - \Delta X_1}{\Delta X_{\max} - \Delta X_1}\right)^{\frac{\Delta X_{\mathrm{ref}} - \Delta X_1}{\lambda}} \mathrm{e}^{-\frac{\Delta X - \Delta X_{\mathrm{ref}}}{\lambda}}$$

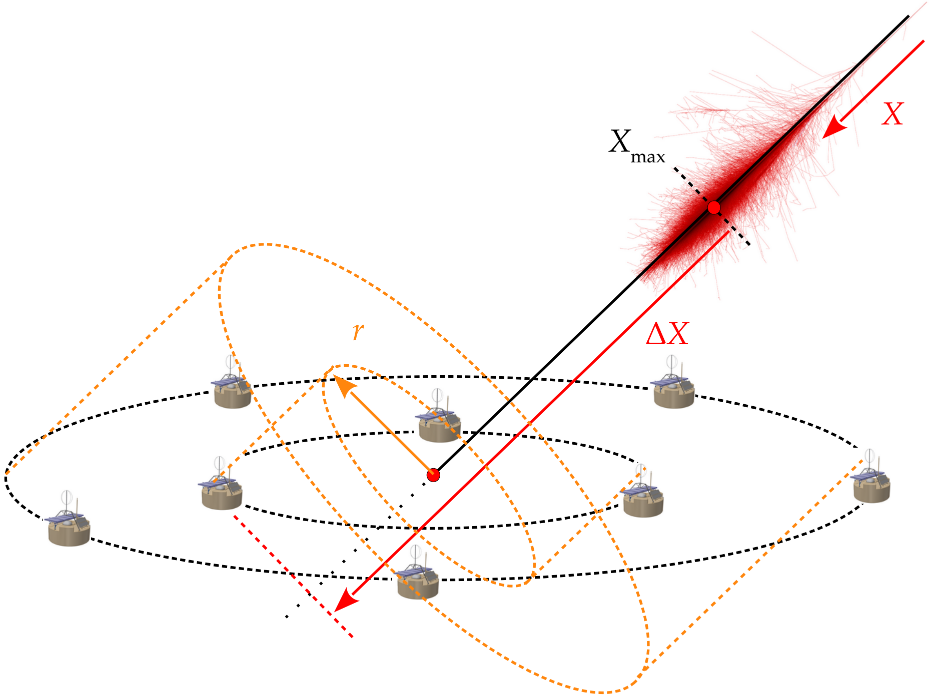
The lateral and longitudinal profile



$$\varrho(r,\Delta X) = \varrho(r)_{\text{ref}} \left(\frac{\Delta X - \Delta X_1}{\Delta X_{\max} - \Delta X_1}\right)^{\frac{\Delta X_{\text{ref}} - \Delta X_1}{\lambda}} \mathrm{e}^{-\frac{\Delta X - \Delta X_{\text{ref}}}{\lambda}}$$

$$\varrho_{\text{ref}}(r) = N \frac{\Gamma(9/2 - s)}{2\pi r_G^2\, \Gamma(s)\Gamma(9/2 - 2s)} \left(\frac{r}{r_G}\right)^{s-2} \left(1 + \frac{r}{r_G}\right)^{s-9/2}$$

The lateral and longitudinal profile

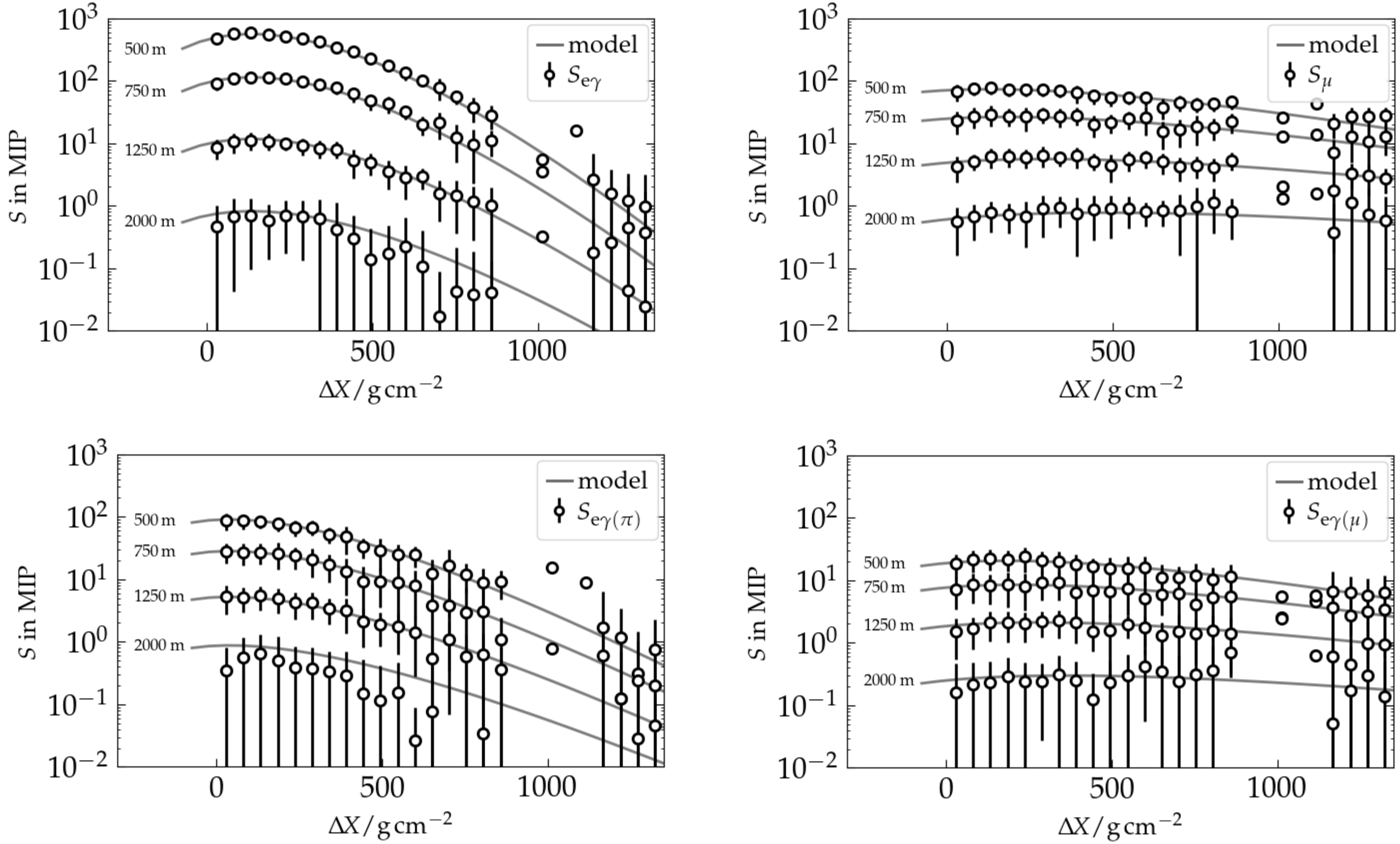


$$\varrho(r,\Delta X) = \varrho(r)_{\mathrm{ref}} \left(\frac{\Delta X - \Delta X_1}{\Delta X_{\max} - \Delta X_1}\right)^{\frac{\Delta X_{\mathrm{ref}} - \Delta X_1}{\lambda}} \mathrm{e}^{-\frac{\Delta X - \Delta X_{\mathrm{ref}}}{\lambda}}$$

$$\varrho_{\mathrm{ref}}(r) = N \frac{\Gamma(9/2-s)}{2\pi r_{\mathrm{G}}^2 \Gamma(s)\Gamma(9/2-2s)} \left(\frac{r}{r_{\mathrm{G}}}\right)^{s-2} \left(1 + \frac{r}{r_{\mathrm{G}}}\right)^{s-9/2}$$

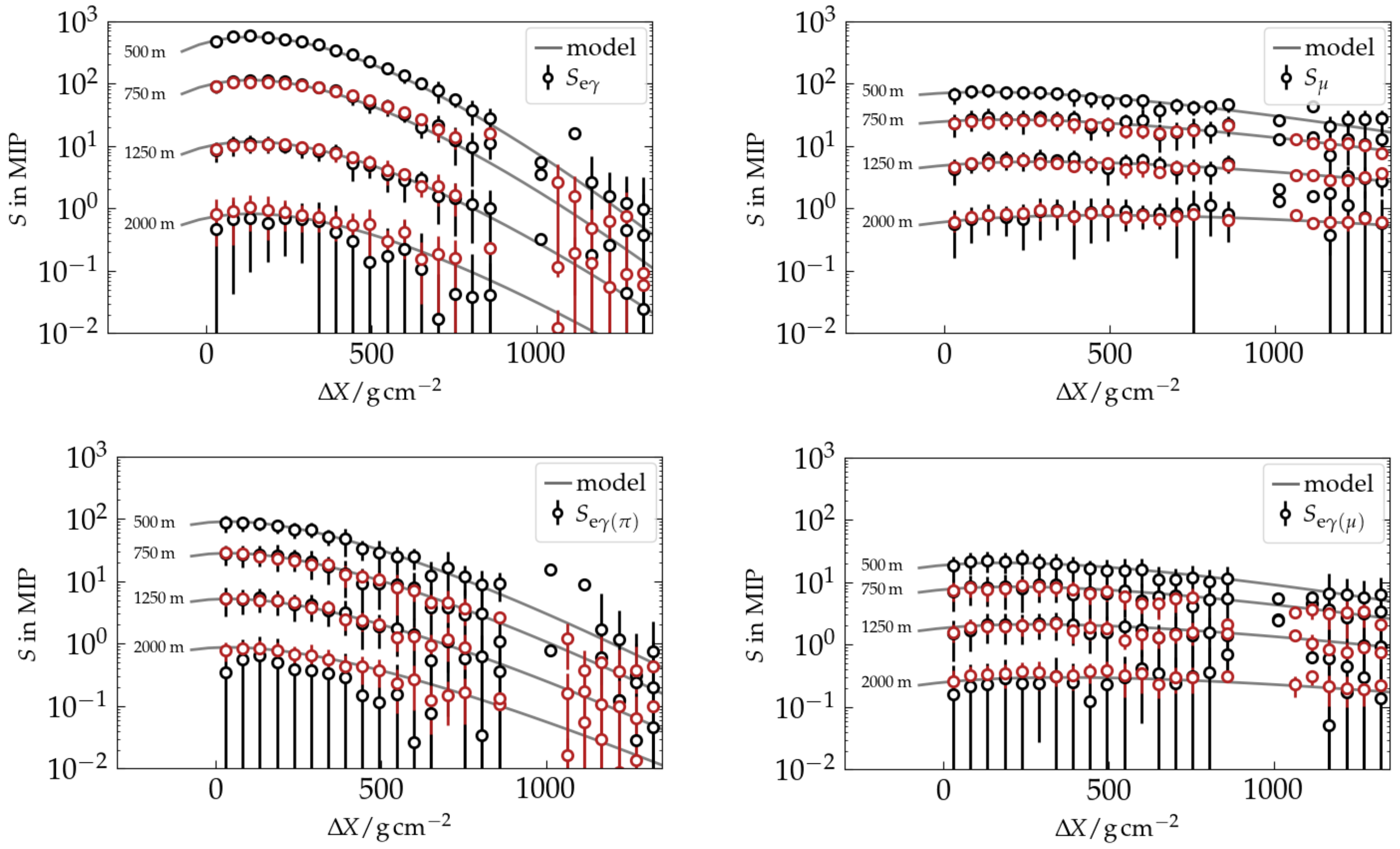
$$N = \left(\frac{E}{10^{19}\,\mathrm{eV}}\right)^{\gamma} N^{19}$$

The lateral and longitudinal profile



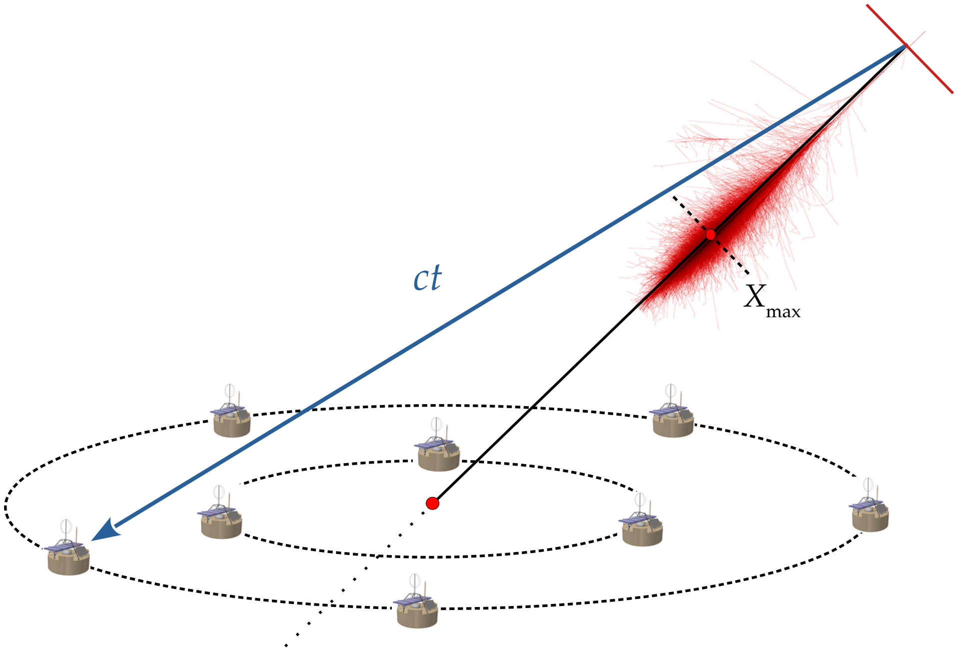
Simulated and predicted signal wrt. ΔX for the four components.
Showers from protons with $E = 10^{19}$ eV (black).

The lateral and longitudinal profile

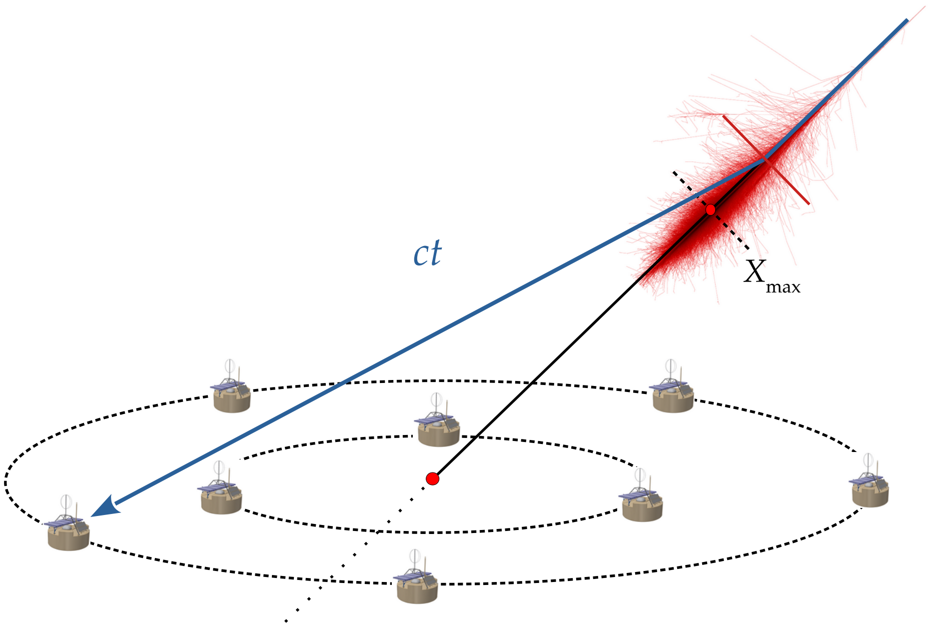


Simulated and predicted signal wrt. ΔX for the four components.
Showers from protons with $E = 10^{19}$ eV (black) and $E = 10^{20}$ eV (red, scaled).

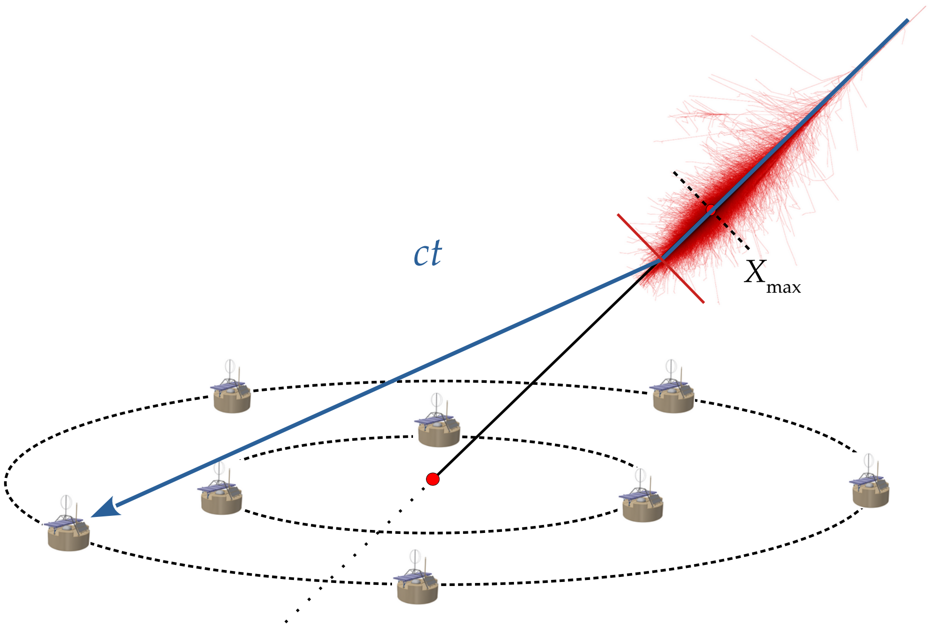
The time dependent signal



The time dependent signal

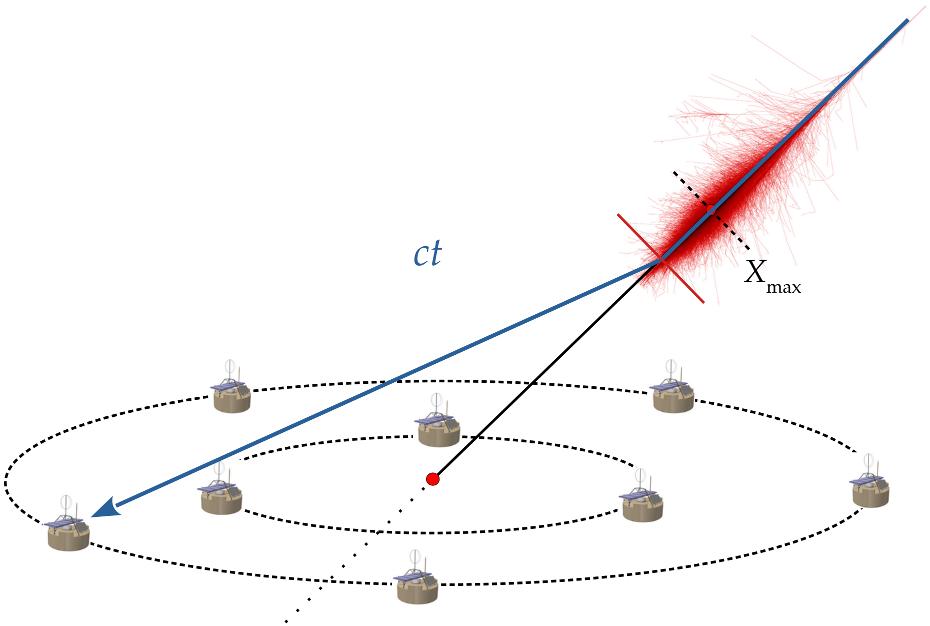


The time dependent signal



$$dN dX \propto dS dt$$

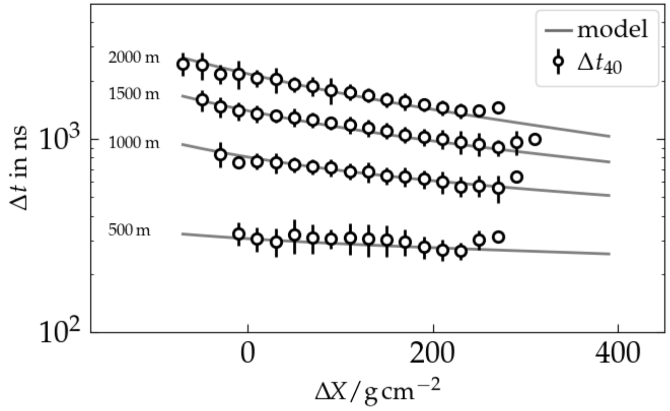
The time dependent signal

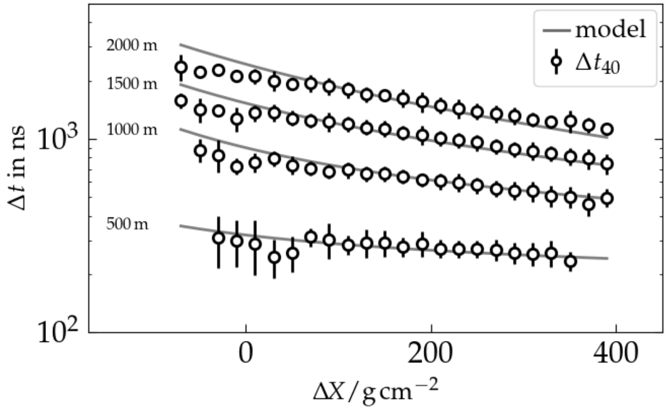


$$\mathrm{d}N\mathrm{d}X \propto \mathrm{d}S\mathrm{d}t$$

$$t_{X_{\mathrm{max}}} - t_{\mathrm{pf}} \simeq t_{40} - t_{\mathrm{pf}} =: \Delta t_{40}$$

The time dependent signal

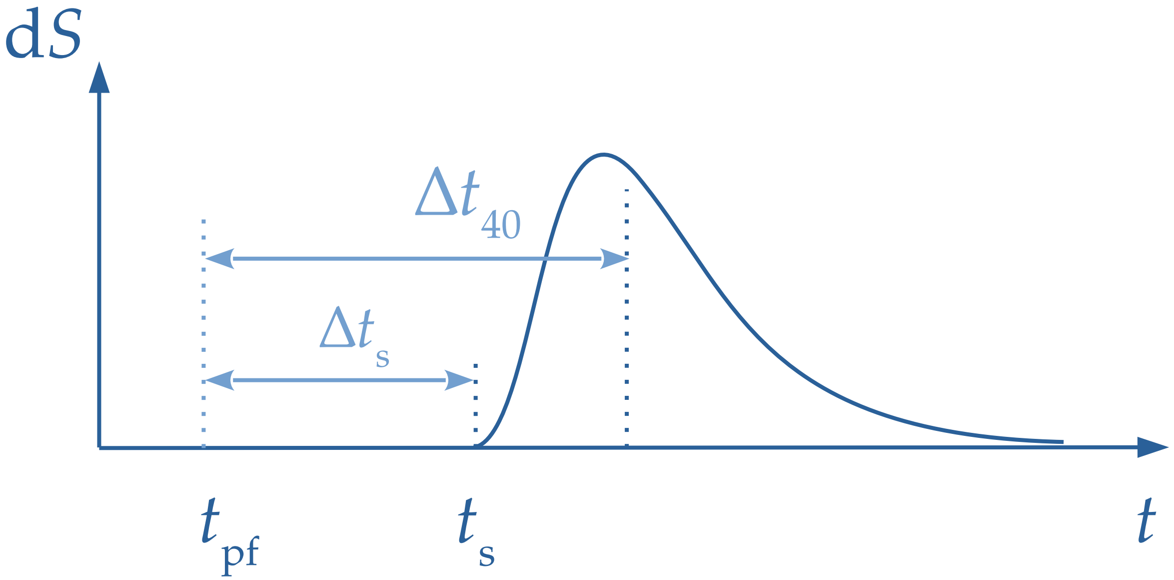




Simulated and predicted values of Δt_{40} for $\theta = 22°$ (*left*) and $\theta = 32°$ (*right*). Showers from protons of $E = 10^{19}$ eV to $E = 10^{20}$ eV.

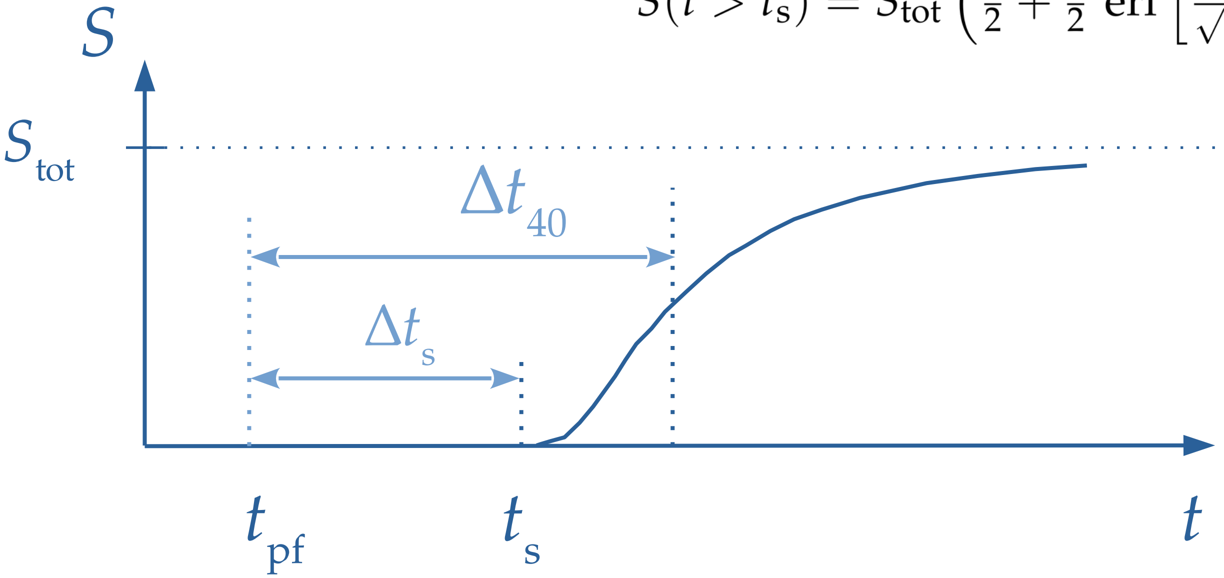
$$c\Delta t_{40} = \sqrt{\sec^2\theta \left(h_\mathrm{s} \ln\left(\frac{X_\mathrm{max}+\Delta X+\delta X}{X_\mathrm{max}}\right)\right)^2 + r^2} - \sec\theta\, h_\mathrm{s} \ln\left(\frac{X_\mathrm{max}+\Delta X+\delta X}{X_\mathrm{max}}\right) + ct_0$$

The time dependent signal



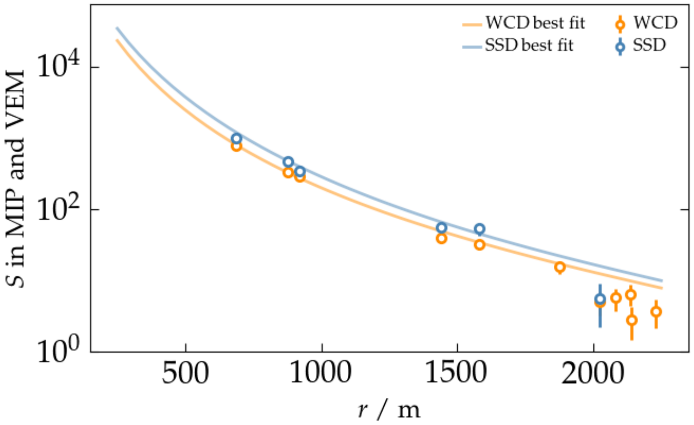
The time dependent signal

$$S(t > t_s) = S_{tot}\left(\frac{1}{2} + \frac{1}{2}\,\mathrm{erf}\left[\frac{1}{\sqrt{2}\,\sigma}\ln\left(\frac{t-t_s}{\Delta t_{40}-\Delta t_s}\right) + \mathrm{erf}^{-1}(2\times 0.4-1)\right]\right)$$



The reconstruction

The reconstruction

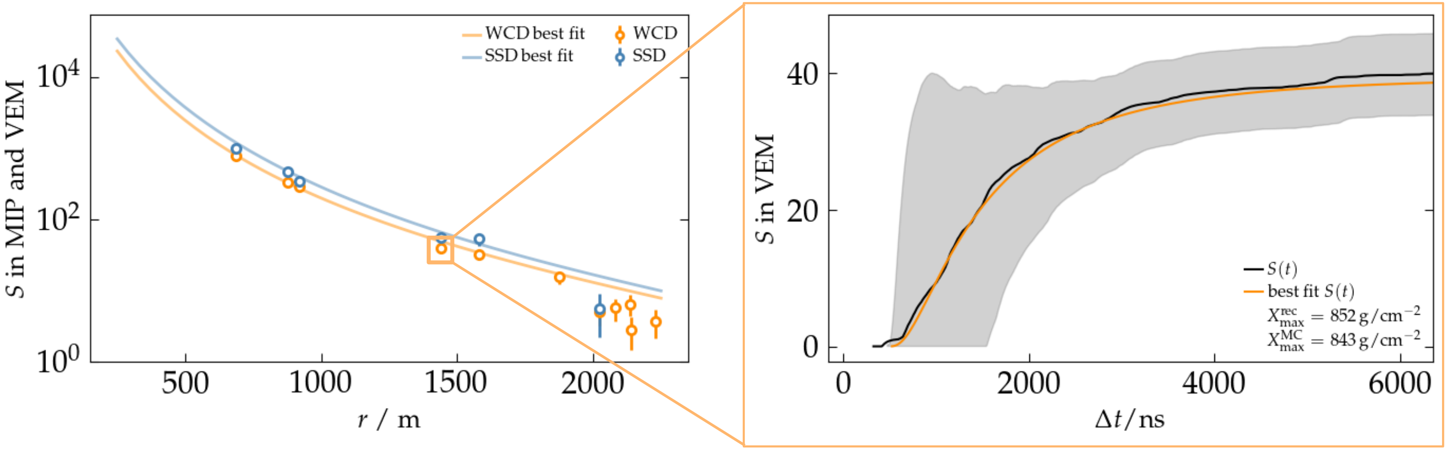


Example of a reconstructed event
$E=10^{19.75}$ eV, $\theta = 28.2°$

best fit values:
$R_\mu = 0.86$
$X_{max} = 852$ g/cm²

MC values:
$R_\mu = 0.74$
$X_{max} = 843$ g/cm²

The reconstruction

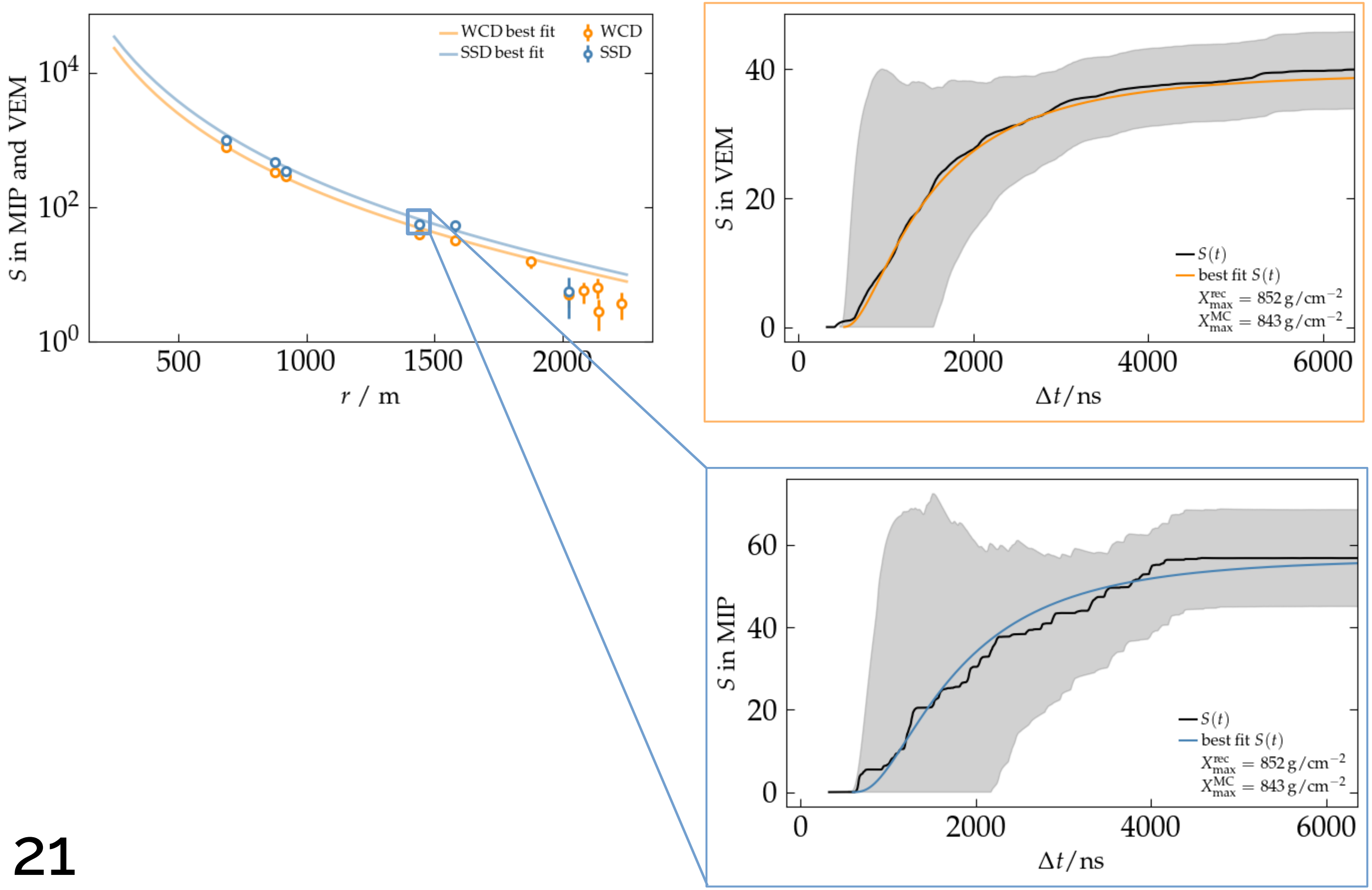


Example of a reconstructed event
$E = 10^{19.75}$ eV, $\theta = 28.2°$

best fit values:
R_μ = 0.86
X_{max} = 852 g/cm²

MC values:
R_μ = 0.74
X_{max} = 843 g/cm²

The reconstruction

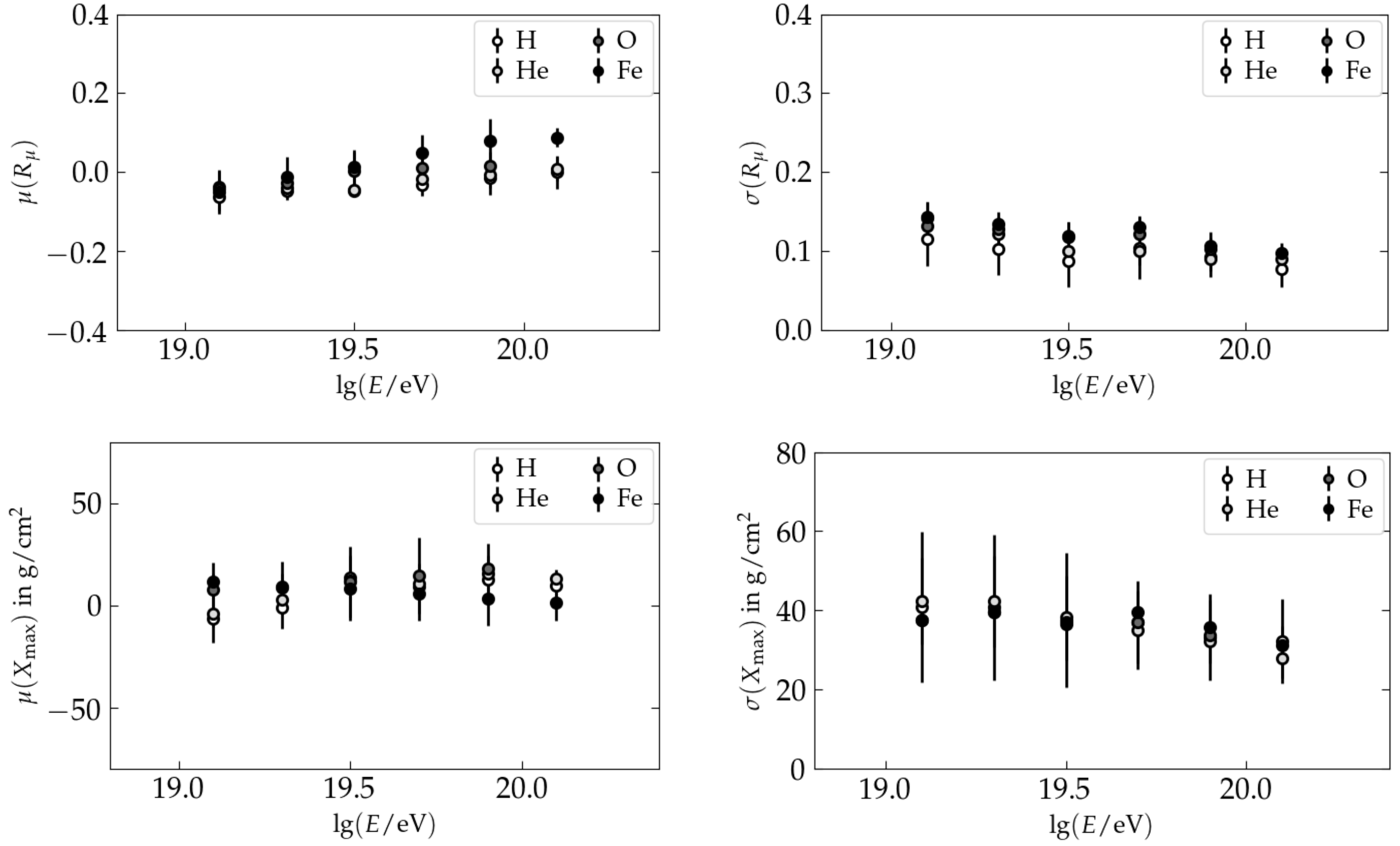


Example of a reconstructed event
$E = 10^{19.75}$ eV, $\theta = 28.2°$

best fit values:
$R_\mu = 0.86$
$X_{max} = 852$ g/cm²

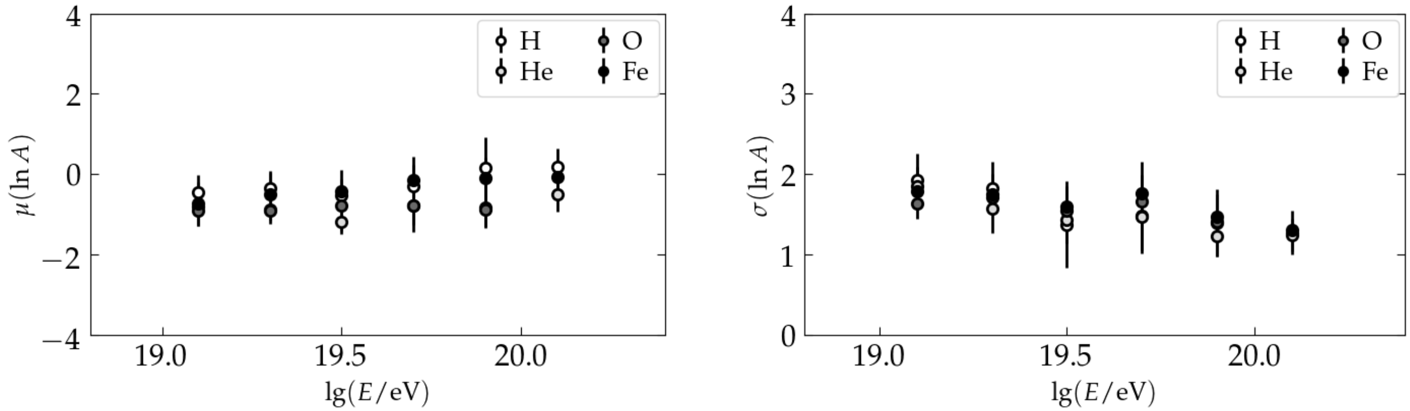
MC values:
$R_\mu = 0.74$
$X_{max} = 843$ g/cm²

The reconstruction



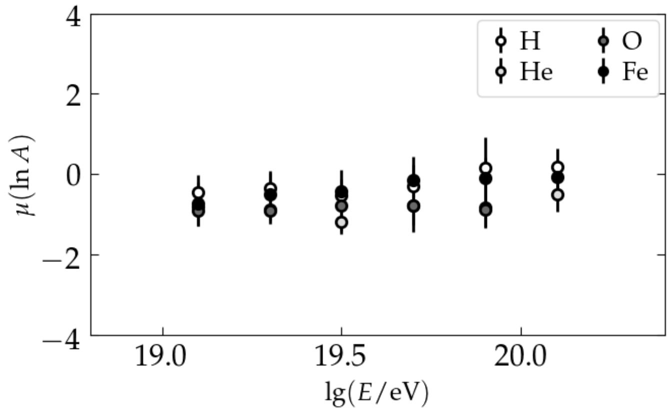
Bias (*left*) and accuracy (*right*) of the reconstruction of R_μ (*top*) and X_{max} (*bottom*), using 16000 simulated showers from different primaries. Results averaged over 10 equidistant bins in zenith angle from 0° to 50°.

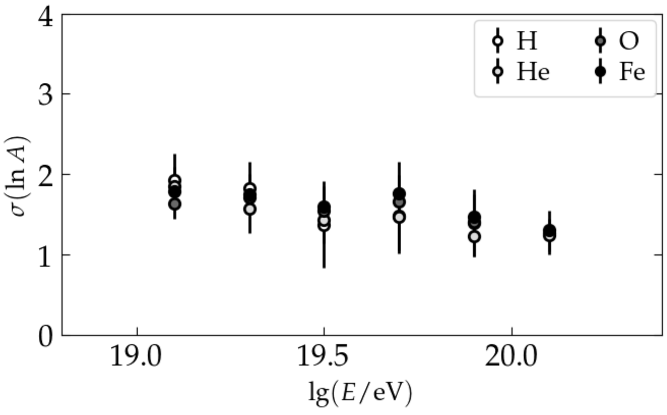
The reconstruction



Bias (*left*) and accuracy (*right*) of the reconstruction of $\ln A$,
using 16000 simulated showers from different primaries. Results averaged over 10 equidistant bins in zenith angle from 0° to 50°.

The reconstruction





Bias (*left*) and accuracy (*right*) of the reconstruction of $\ln A$,
using 16000 simulated showers from different primaries. Results averaged over 10 equidistant bins in zenith angle from 0° to 50°.

Max Stadelmaier
Markus Roth, David Schmidt, Darko Veberič

Karlsruher Institut für Technologie
UNIVERSIDAD NACIONAL DE SAN MARTÍN
HIRSAP
Double Doctoral degree in Astrophysics DDAp

max.stadelmaier@kit.edu