Spectral Parameterization of GCR observations and reconstruction of solar modulation parameters derived from the Convection-Diffusion approximation

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Overview

- Analytical approximations of the Parker transport equation.
- Fitting of 1 AU monthly proton fluxes.
- Comparison of fit statistics, derived modulation parameters and the effective diffusion coefficient parameters.
- Conclusions based on the results.

Solar Modulation:

- Parker transport equation (Gleeson & Axford [1968])
- Lowest-order approximate solutions (For mathematical description, see e.g. Caballero-Lopez & Moraal [2004], Moraal [2011], Mosotho and Strauss [2021], amongst others.):
  1. Force-Field Approximation (FFA)

     $$\frac{j_e(r,\mathrm{P})}{j_*(r_*,\mathrm{P}_*)}^{1} = \left[\frac{\mathrm{P}}{\mathrm{P}_*}\right]^2 \qquad (1)$$

     with P_* calculated for the special case of relativistic particles.
  2. Convection-Diffusion Approximation (CDA)

     $$\frac{j_e(r,\mathrm{P})}{j_*(r_*,\mathrm{P}_*)} = \frac{1}{e^M} \qquad (2)$$

     where

     $$M = M(r,\mathrm{P},t) = \int_r^{r_*} \frac{\mathrm{V}(t)}{\kappa} dr \qquad (3)$$

[1] The "e" and "$*$" denotes the 1 AU and LIS references.

- Input parameters:
  1. Double power-law LIS:

$$j_*(\mathrm{T}) = j_0 \left(\frac{\mathrm{T}}{\mathrm{T}_0}\right)^{\mu_0} \left(\mu_1 \left(\frac{\mathrm{E}_0}{\mathrm{T}_0}\right) + \left(\frac{\mathrm{T}}{\mathrm{T}_0}\right)^{\mu_2}\right)^{\mu_3} \tag{4}$$

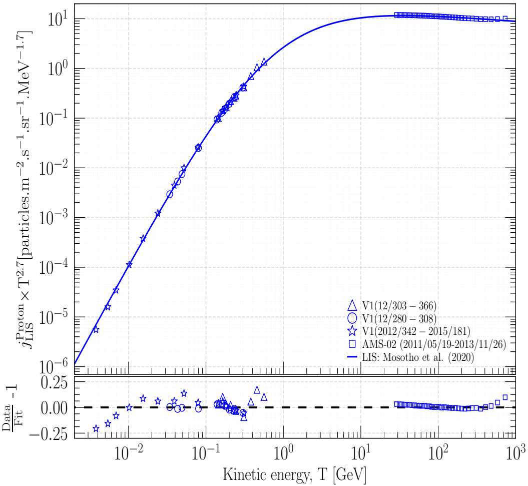
     where $\mathrm{T}_0 = 1$ GeV, while j_0 is given as 20.98 in units of $\mathrm{particles/m^2/s/sr/MeV}$. The μ constant values are given as $\mu_0 = 0.179$, $\mu_1 = 0.777$, $\mu_2 = 0.795$ and $\mu_3 = 3.7764$.
  2. Effective diffusion coefficient, $\kappa = \kappa(r, \mathrm{P}, t)$:

$$\kappa = \beta^{\eta(t)} \kappa_0(r,t) \kappa_{\mathrm{P}}(\mathrm{P},t), \text{ with } \kappa_{\mathrm{P}}(\mathrm{P},t) = \mathrm{P}^{\gamma(t)} \tag{5}$$

  3. In Eq. (1) and (2) : $\int_{\mathrm{P}}^{\mathrm{P}_*} \frac{\beta^{\eta(t)} \mathrm{P}^{\gamma(t)}}{\mathrm{P}} d\mathrm{P} = \int_r^{r_*} \frac{\mathrm{V}(r)}{3\kappa_0} dr \equiv \phi(t)$, while $M(t) = \Re(t) \cdot \beta^{-\eta(t)} \cdot \mathrm{P}^{-\gamma(t)}$, respectively.

Example: LIS Parameterization

LIS: AMS-02 and Voyager-1 fit



- Voyager-1 data used to deduce LIS at low energies.
- At neutron montor energies the LIS is unkown.
- To avoid residual solar modulation, the AMS-02 data was only fitted above 30 GeV.
- As shown in the mini panel, the LIS can reproduce observation within $\pm 25\%$.

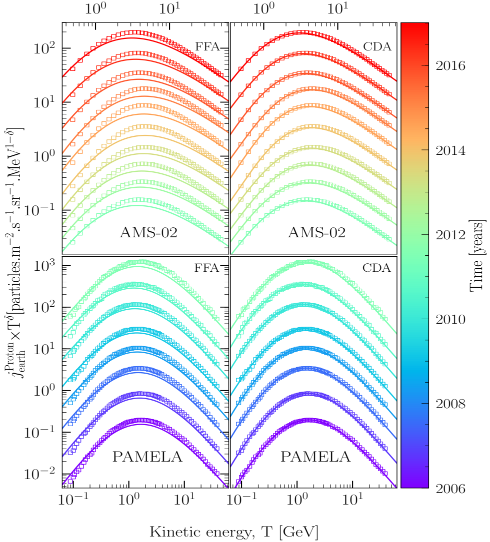


Figure: **Top panels:** AMS-02 measured GCR proton spectra and the fit approximations (solid lines), while the PAMELA spectra and fits are shown in the **bottom panels**. Data credit https://www.ssdc.asi.it/..

Pearson's reduced chi-squared [χ_r^2] :

$$\chi_r^2 = \frac{\chi^2}{ndf} \equiv \frac{1}{ndf} \sum_i \frac{\left(j_i^{\mathrm{O}} - j_i^{\mathrm{F}}\right)^2}{j_i^{\mathrm{F}}}, \tag{6}$$

where i is the binning index, j_i^{O} and j_i^{F} are the observed TOA fluxes and its associated fit approximation for the i^{th} data bin, while the number of degree of freedom $(ndf) =$ (number of data points) - (number of free parameters).

In this work, we calculate χ_r^2 for two scenarios:

- The time-dependent χ_r^2 is calculated for each Carrington rotation-averaged spectra.
- An energy-dependent χ_r^2 is calculated by using the fit statistic for each kinetic energy bin and averaging this over all time periods.

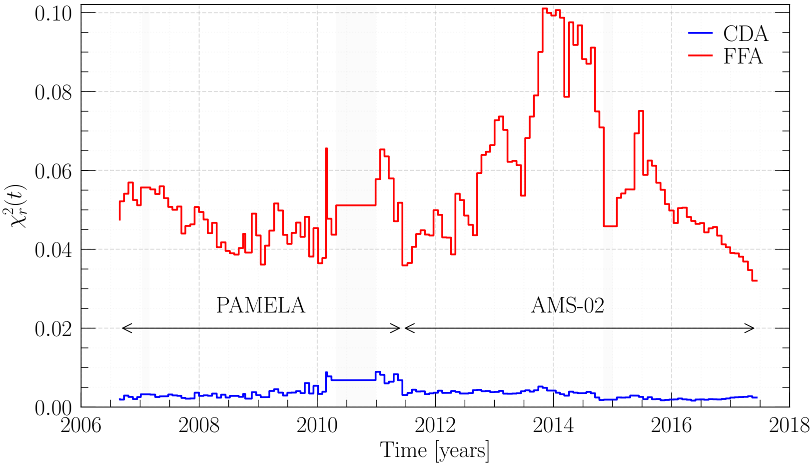


Figure: χ_r^2 values obtained through fitting the time-dependent spectra.

Fit statistics [2]

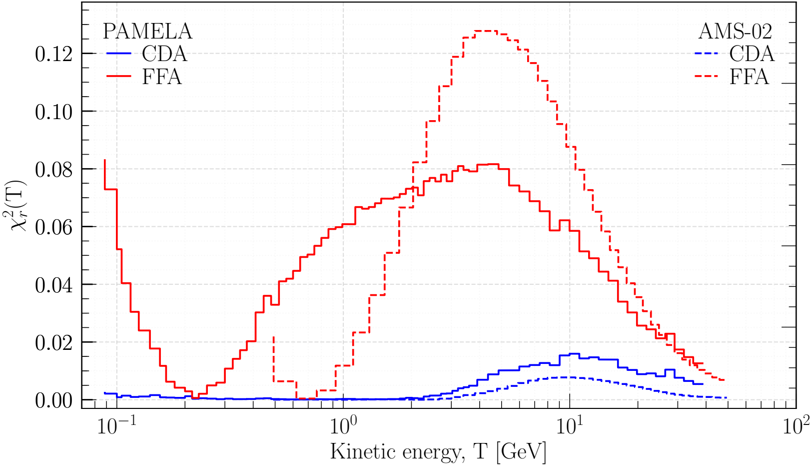


Figure: Here, χ_r^2 is given as a function of energy and averaged over all time periods.

Modulation parameters

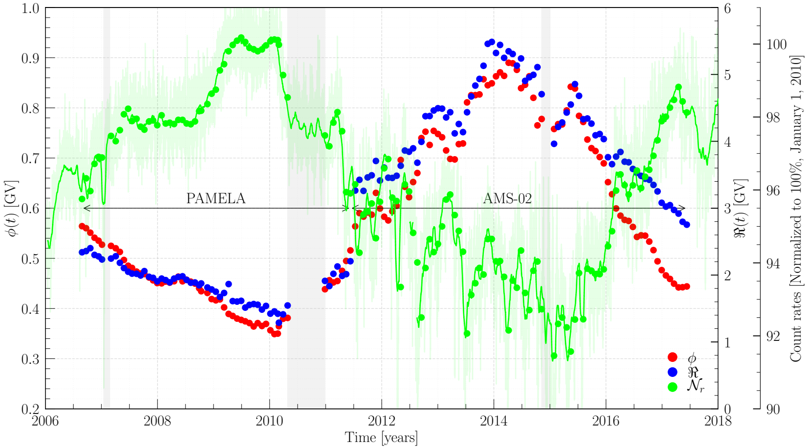


Figure: Best–fit solar modulation parameters ϕ and $\Re$ compared to NM counts rates $\mathcal{N}_r$ variations measured by the Hermanus station (data credit http://www.nwu.ac.za/neutron-monitor-data).

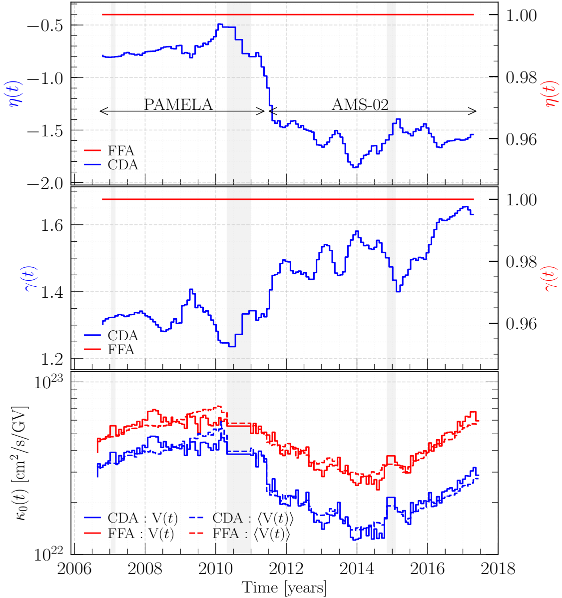


Figure: **Top & middle panels:** CDAs temporal evolution of η and γ (in the FFA they're fixed). **Bottom panel:** κ_0 estimated for long-term and Carrington averages.

Effective Diffusion Coefficient

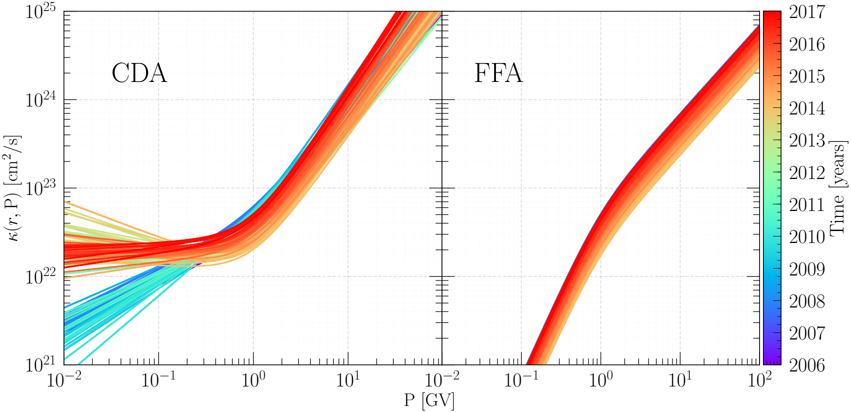


Figure: Evolution of the estimated proton diffusion coefficient, κ, at Earth for different times as a function of rigidity.

Conclusions

- We have validated the CDA over an $\sim$11 years solar cycle using the PAMELA and AMS-02 observations.
- By a direct comparison of the fit-to-data spectra -OR- simply by looking at the reduced chi-squared, the CDA appears to be significantly more accurate than the FFA.
- Due to the CDA's high accuracy, it offers a better alternative to the FFA for parameterizing the 1 AU GCR flux.
- Accurately characterizing the GCR fluxes can be of help in the interpretation of aviation dosimetric calculations.

Thank you for your attention !

“Unless you believe, you will not understand”
-St. Aurelius Augustinus Augustine

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The use and validation of the Convection-Diffusion approximation in cosmic-rays modulation studies

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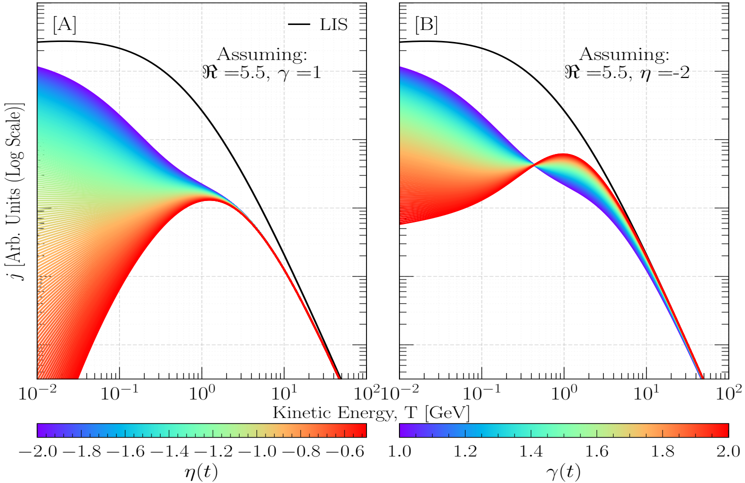
CDA : Fit Demonstration



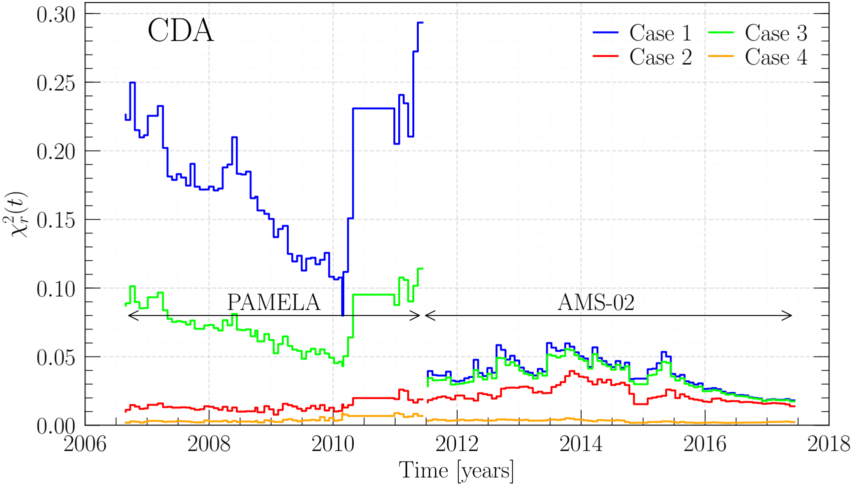
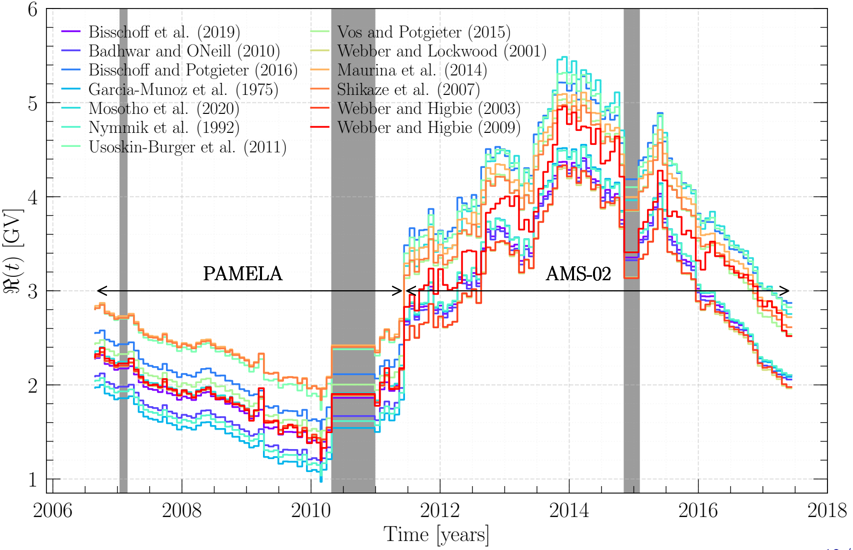
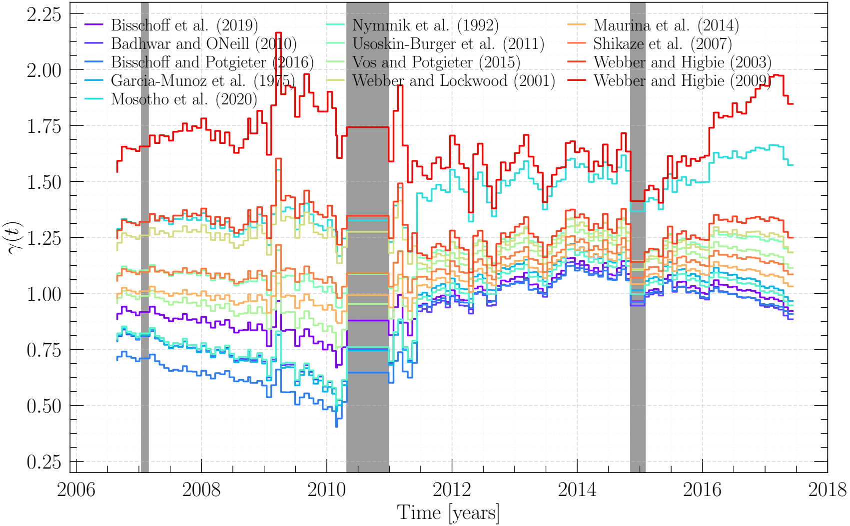


Figure: For Case 1 $\eta = 1$ and $\gamma = \gamma(t)$, Case 2 $\gamma = 1$ and $\eta = \eta(t)$, Case 3 $\gamma(t) = \eta(t)$ and Case 4 $\gamma = \gamma(t)$ and $\eta = \eta(t)$.

CDA : Preliminary Tests



CDA : Preliminary Tests



CDA : Comparing γ and η