Local Turbulence and the Dipole Anisotropy of Galactic Cosmic Rays

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In collaboration
with Markus Ahlers

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The Niels Bohr International Academy

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VILLUM FONDEN

Differential flux: $$\Psi_i = \frac{dN_i}{dE\,dT\,d\Omega\,dS}$$

Energy

Composition

Direction

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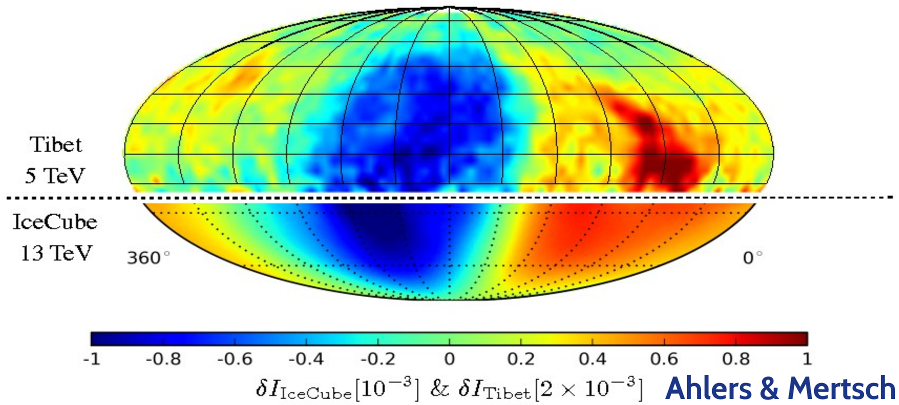
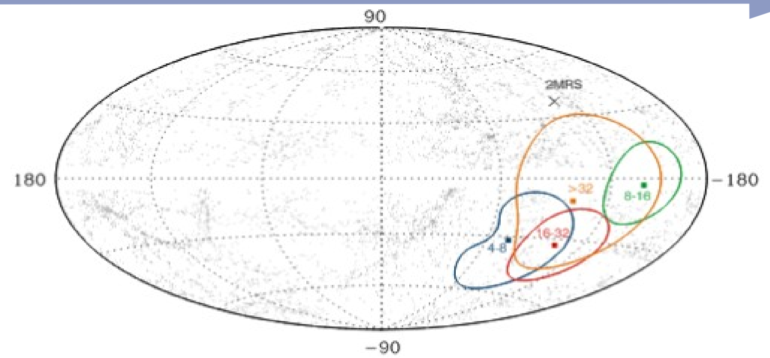
TeV

EeV



Ahlers & Mertsch PPNP (2016)



Auger collab. ApJ 868 (2018)

Introduction: dipole anisotropy of cosmic rays

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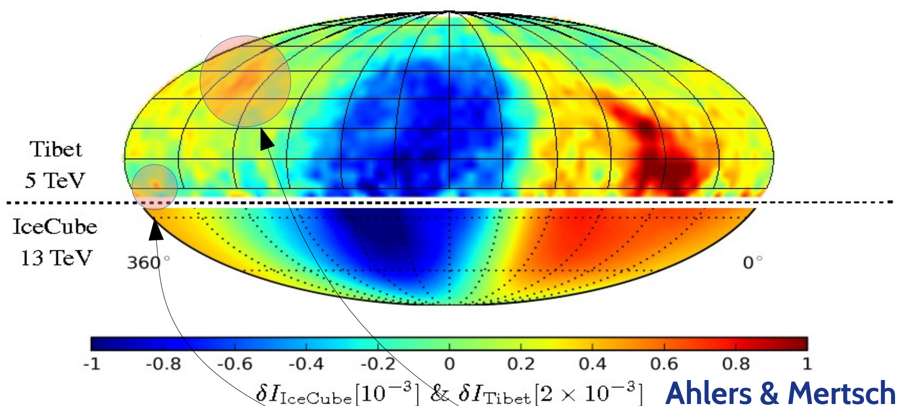
Energy **Composition** **Direction**

TeV → **EeV**



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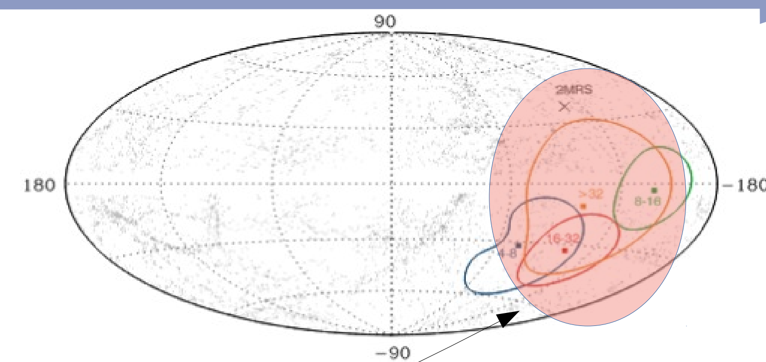

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Multi-scale anisotropies

→ In the following we focus on the dipole anisotropy

Data

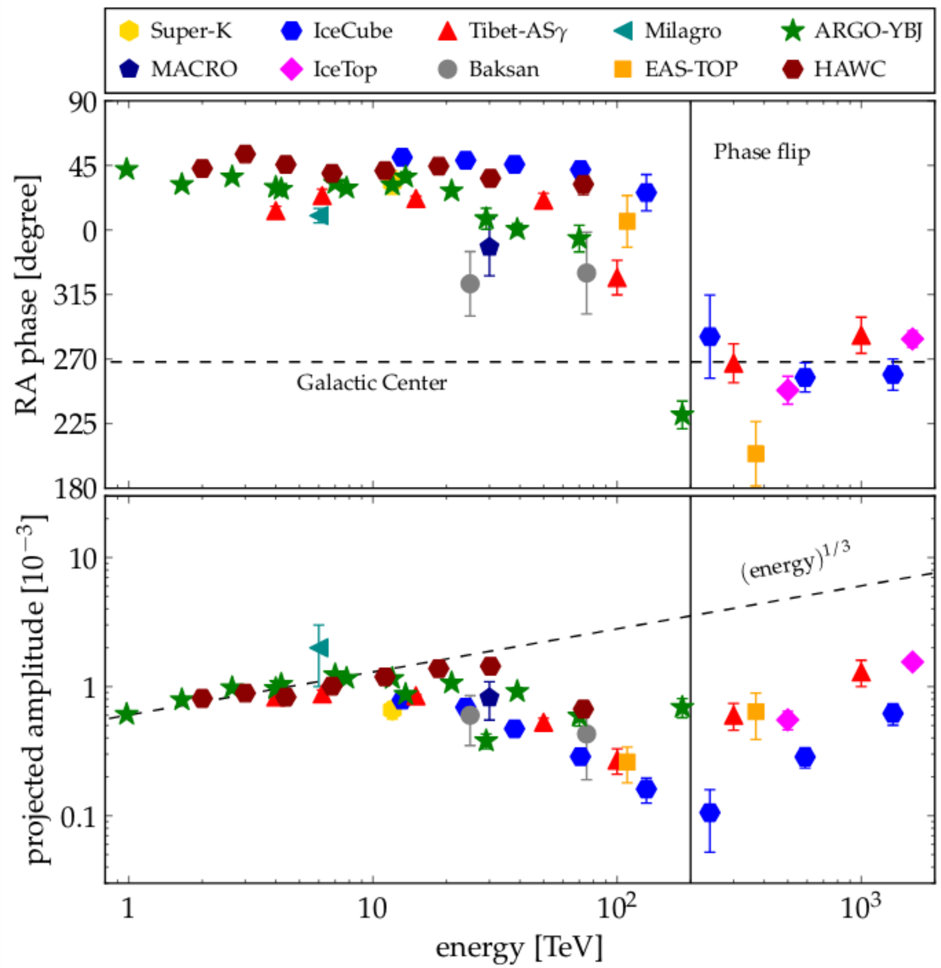
→ Relative intensity can be decomposed as:

$$I(\Omega) = 1 + \boldsymbol{\delta} \cdot \boldsymbol{n}(\Omega) + \mathcal{O}(Y_{l>1})$$

→ CR observatories sensitive to 2 param.

→ Small dipole anisotropy of GCRs

→ Rapid change of the phase & amplitude with E



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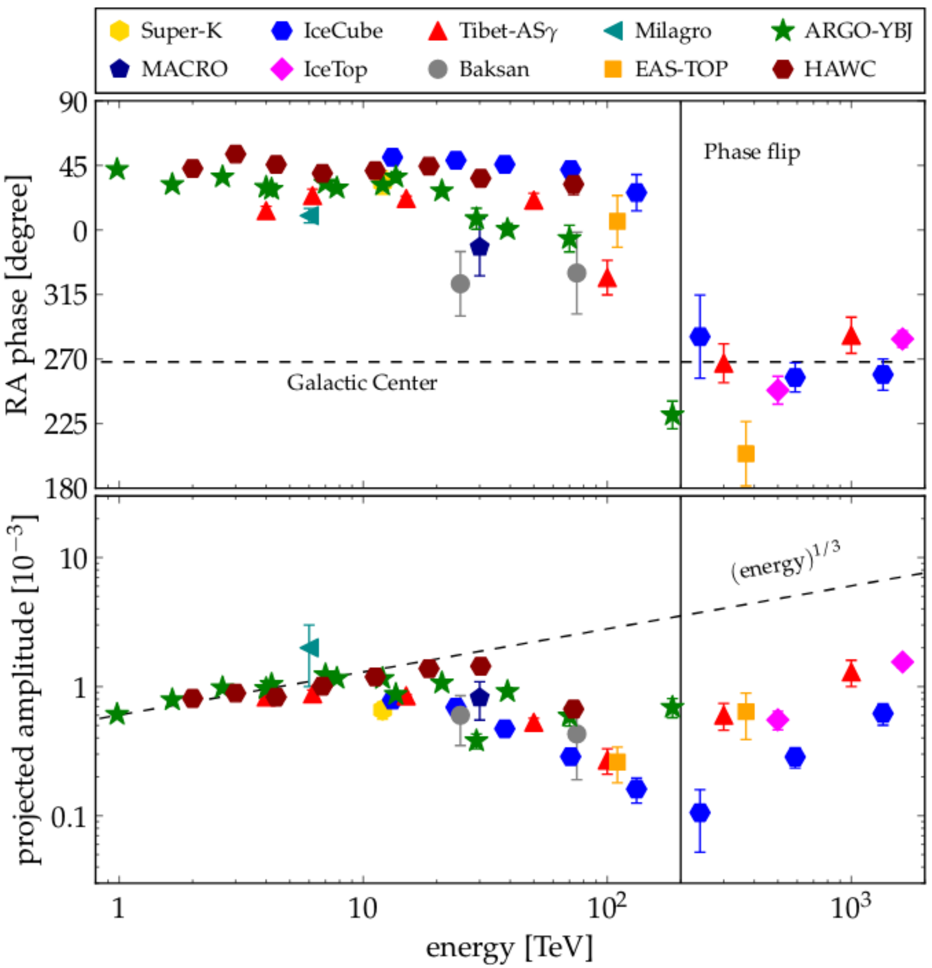

Interpretation

$$\boldsymbol{\delta} \propto \boldsymbol{j}_{\mathrm{CR}}$$

→ Compton Getting effect?
Small in the local standard of rest

→ Diffusion approximation

Fick's law: $\boldsymbol{j}_{\mathrm{CR}} = -\boldsymbol{K} \cdot \nabla\Psi$

Energy dependence at odd with diffusion

Depends on:
- Distribution of sources and halo geometry halo?
- Structure of local magnetic field?

→ **Both!**

Introduction: dipole anisotropy of cosmic rays

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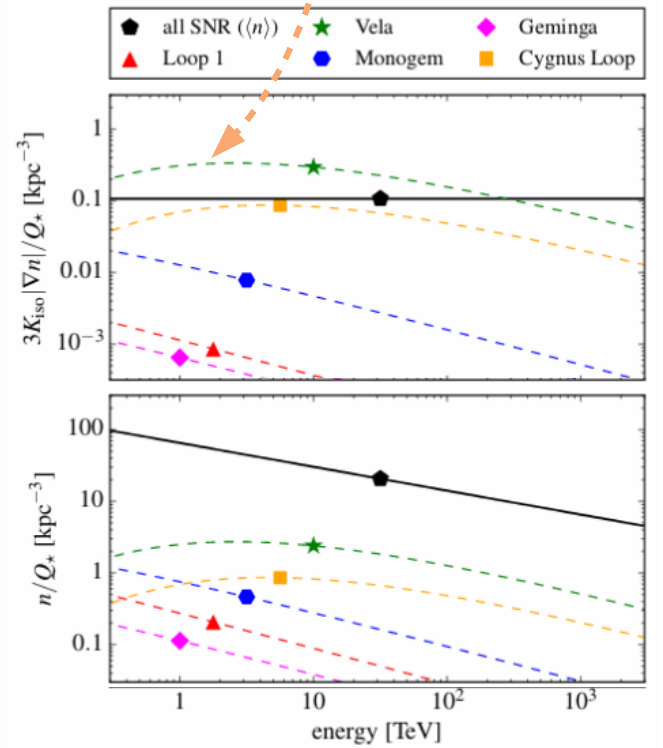
$$j_{\mathrm{CR}} = -\boldsymbol{K} \cdot \nabla\Psi$$

Ahlers PRL (2016)

Local sources may dominate the dipole



→ Evolution of the dipole direction with E

See also e.g. Kumar & Eichler APJ (2014)
Bouyahiaoui+ JCAP (2019)

Introduction: dipole anisotropy of cosmic rays

→ Both!

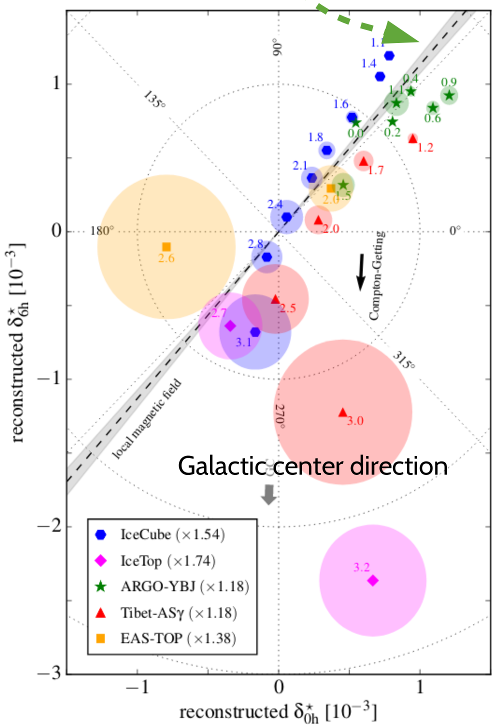
$$\boldsymbol{j}_{\rm CR} = -\boldsymbol{K} \cdot \nabla\Psi$$

Ahlers PRL (2016)

$$K_{ij} = b_i b_j \kappa_{\parallel} + (\delta_{ij} - b_i b_j)\kappa_{\perp}$$

Local magnetic field direction

Local sources may dominate the dipole



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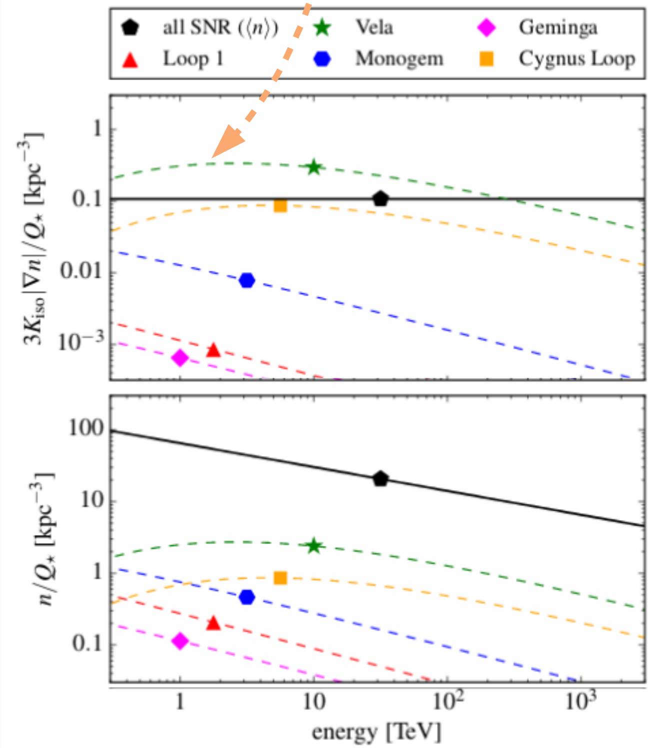

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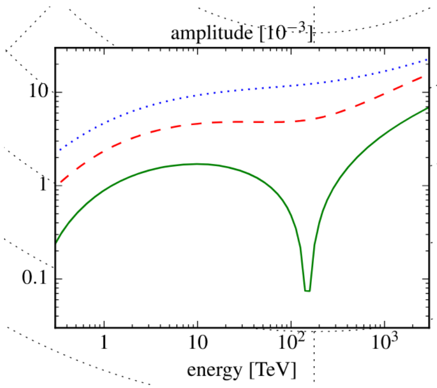

→ Projection of the dipole on the local magnetic field

See also e.g. Funk & Mertsch PRL (2013)

→ Evolution of the dipole direction with E

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Introduction: dipole anisotropy of cosmic rays

How does behave the CR dipole in **isotropic turbulence**?

$$\boldsymbol{\delta} \propto \boldsymbol{j}_{\mathrm{CR}} = -\boldsymbol{K} \cdot \nabla\Psi$$

→ We naively write: $K_{ij} = \delta_{ij}\kappa_{\mathrm{iso}}$ $\kappa_{\mathrm{iso}} = \lim_{\tau\to\infty} \langle \Delta\mathbf{r}^2(\tau)\rangle_{\mathrm{B}}/6\tau$

Does it mean that the anisotropy follow the gradient direction?

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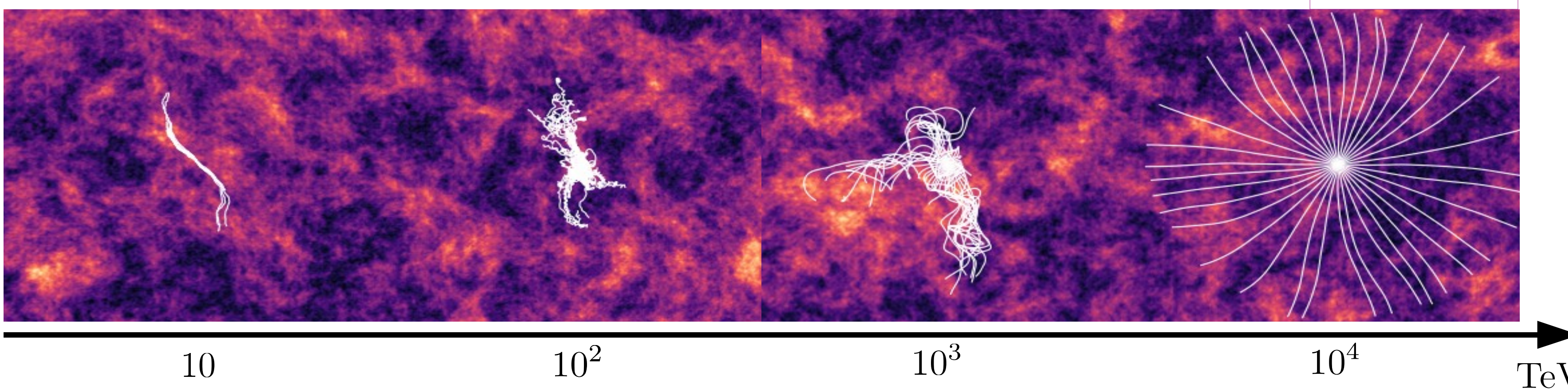
Test-particle simulations: backtracking in **isotropic turbulence**:

$l_c = 2\,\mathrm{pc}$
$B_{\mathrm{rms}} = 4\mu\mathrm{G}$



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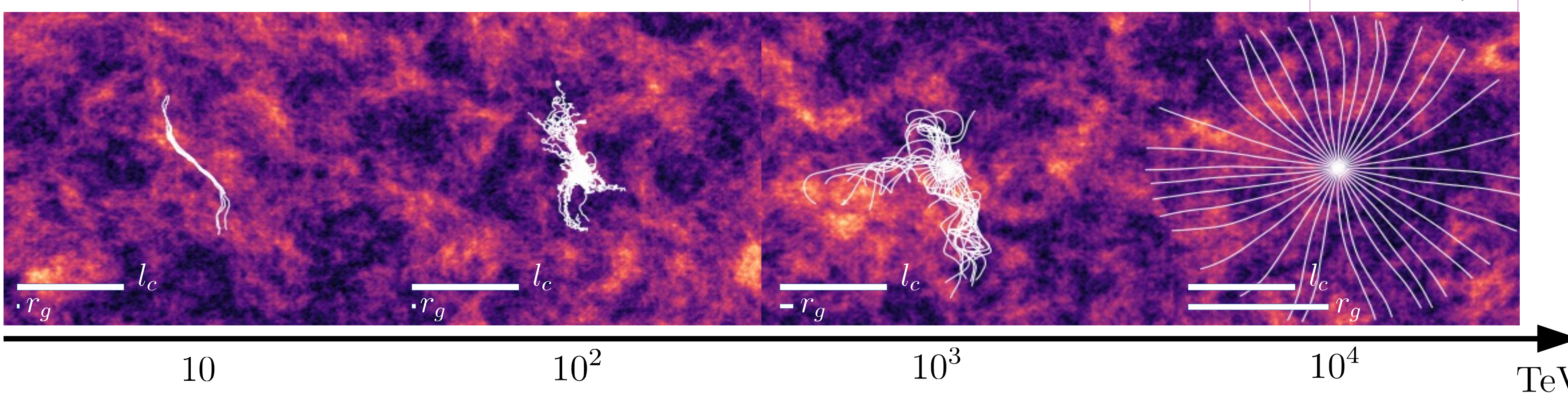

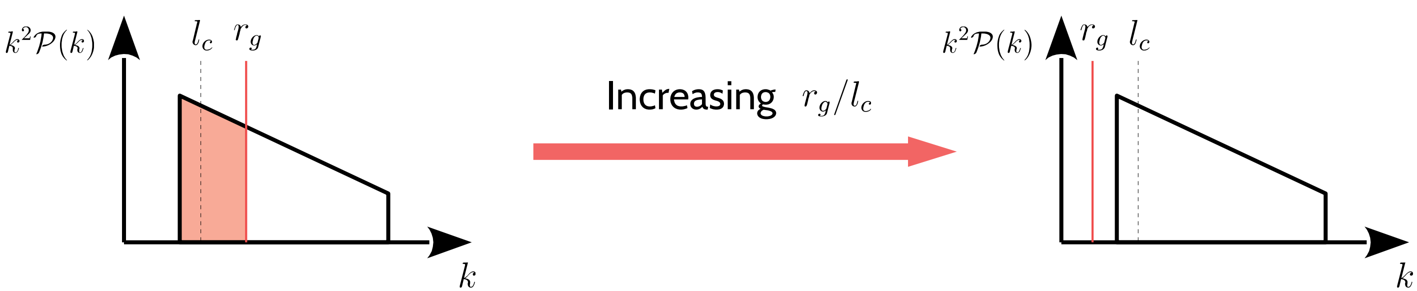

→ At low energies particles stream along the local magnetic field

Formalism

Angular power spectrum of CR arrival directions:

$$\frac{C_\ell}{4\pi} \simeq \int \frac{\mathrm{d}\widehat{\mathbf{p}}_1}{4\pi} \int \frac{\mathrm{d}\widehat{\mathbf{p}}_2}{4\pi} P_\ell(\widehat{\mathbf{p}}_1 \widehat{\mathbf{p}}_2) \lim_{\tau\to\infty} \left(\Delta r_{1i}(-\tau)\Delta r_{2j}(-\tau)\right) \frac{\partial_i n \partial_j n}{n^2}$$

Ahlers & Mertsch AJL (2015)

CR dipole power:

Dipole depends on the eigen values of S

$$\frac{C_1}{4\pi} \simeq \mathrm{S}_{ij} \frac{\partial_i n \partial_j n}{n^2}$$

with $\mathbf{S} \equiv \mathcal{K}^T \mathcal{K}$ and $\mathcal{K}_{ij} \equiv \lim_{\tau\to\infty} \langle \widehat{p}_i(0) \Delta r_j(-\tau) \rangle_\Omega$

Coincides with the TKG definition if:

Ω ↔ Ensemble B

→ Study of the diffusion tensor with test-particle simulations

Simulation set up

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Nested grid method Giacinti et al. PRL (2012)
→ Large dynamical range > 10^5

Customized CRpropa version
→ Integration with Boris-Push algorithm

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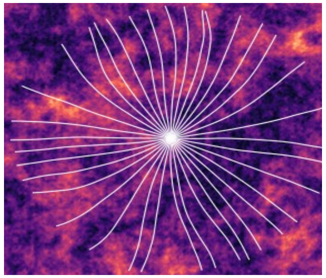
- Backtrack particles with unif. distributed orientations
- 150 random B-field realizations



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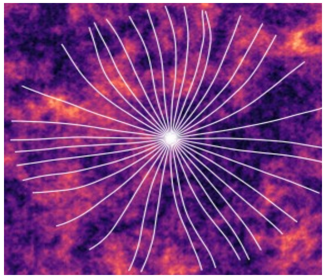
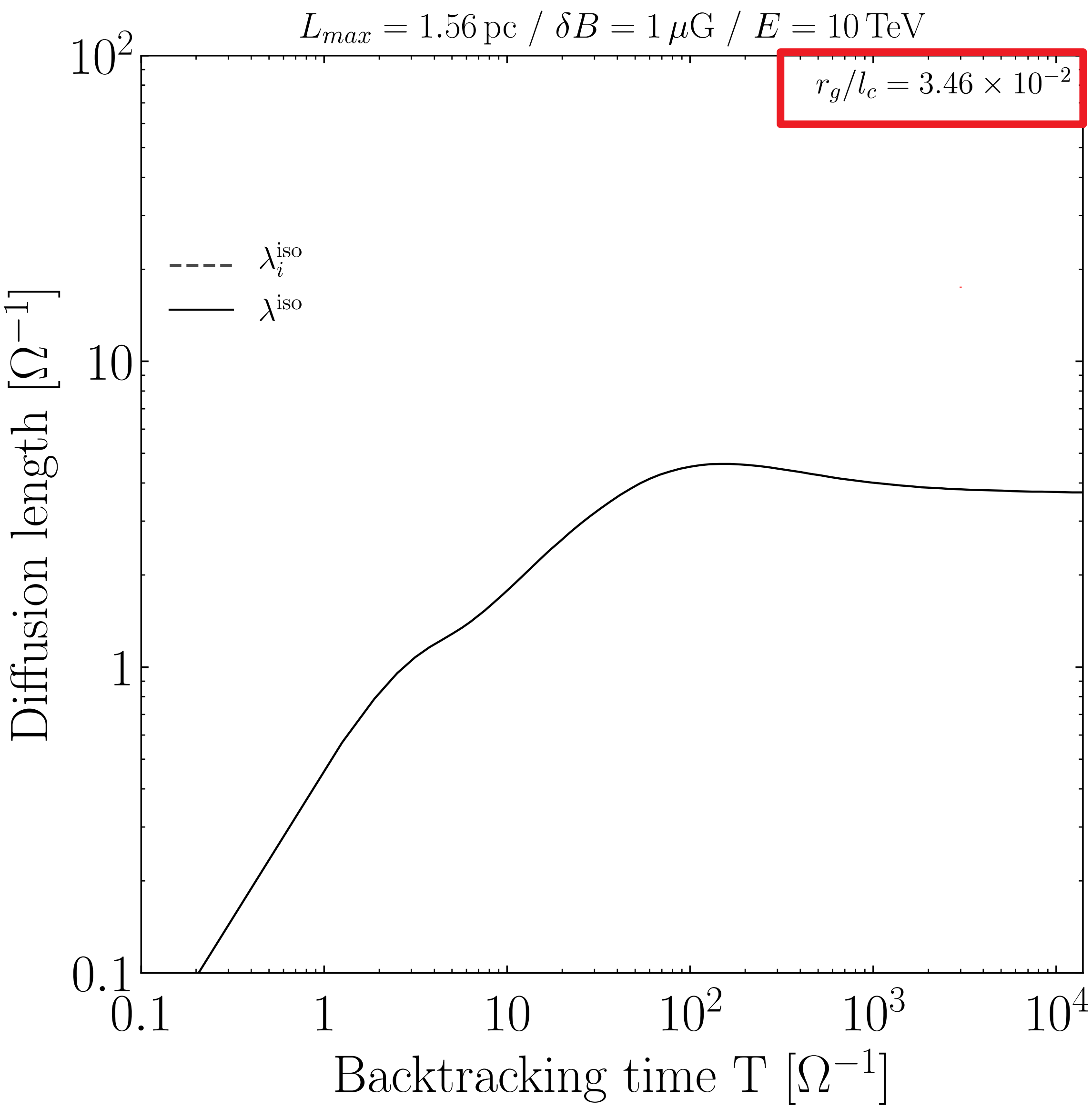


Compute $\mathcal{K}$ and $\mathbf{S} \equiv \mathcal{K}^T \mathcal{K}$
Extract the three eigen values

Results

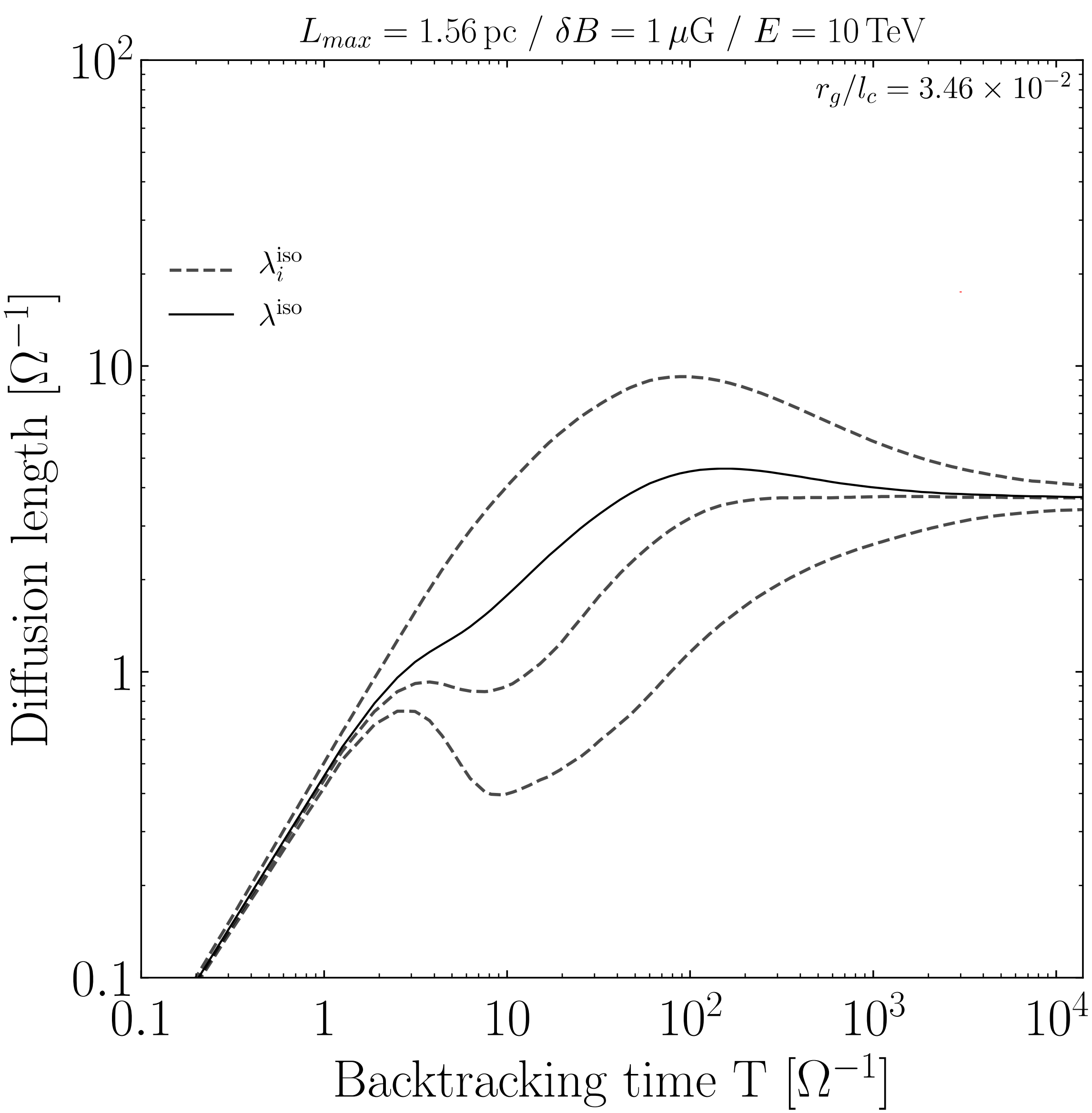


Conventional diffusion tensor

$$\kappa_{lm}(T) = \langle \Delta r_l(-T)\Delta r_m(-T)\rangle_\Omega / 2T$$

→ Isotropic diffusion length

$$\lambda^{\mathrm{iso}} = \frac{1}{3}\langle \mathrm{Tr}(\kappa_{lm}(T))\rangle_{\mathrm{B}}$$



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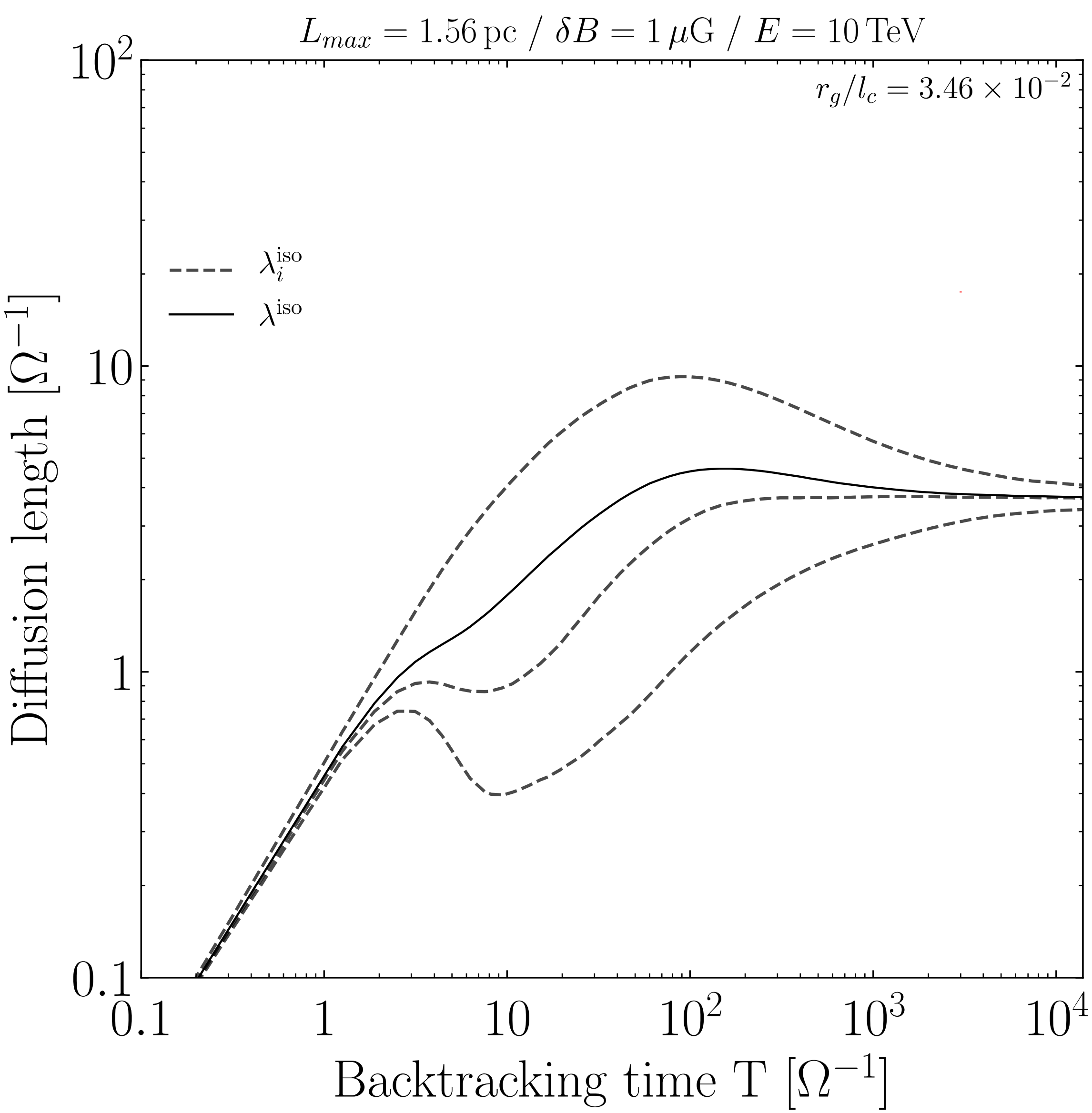

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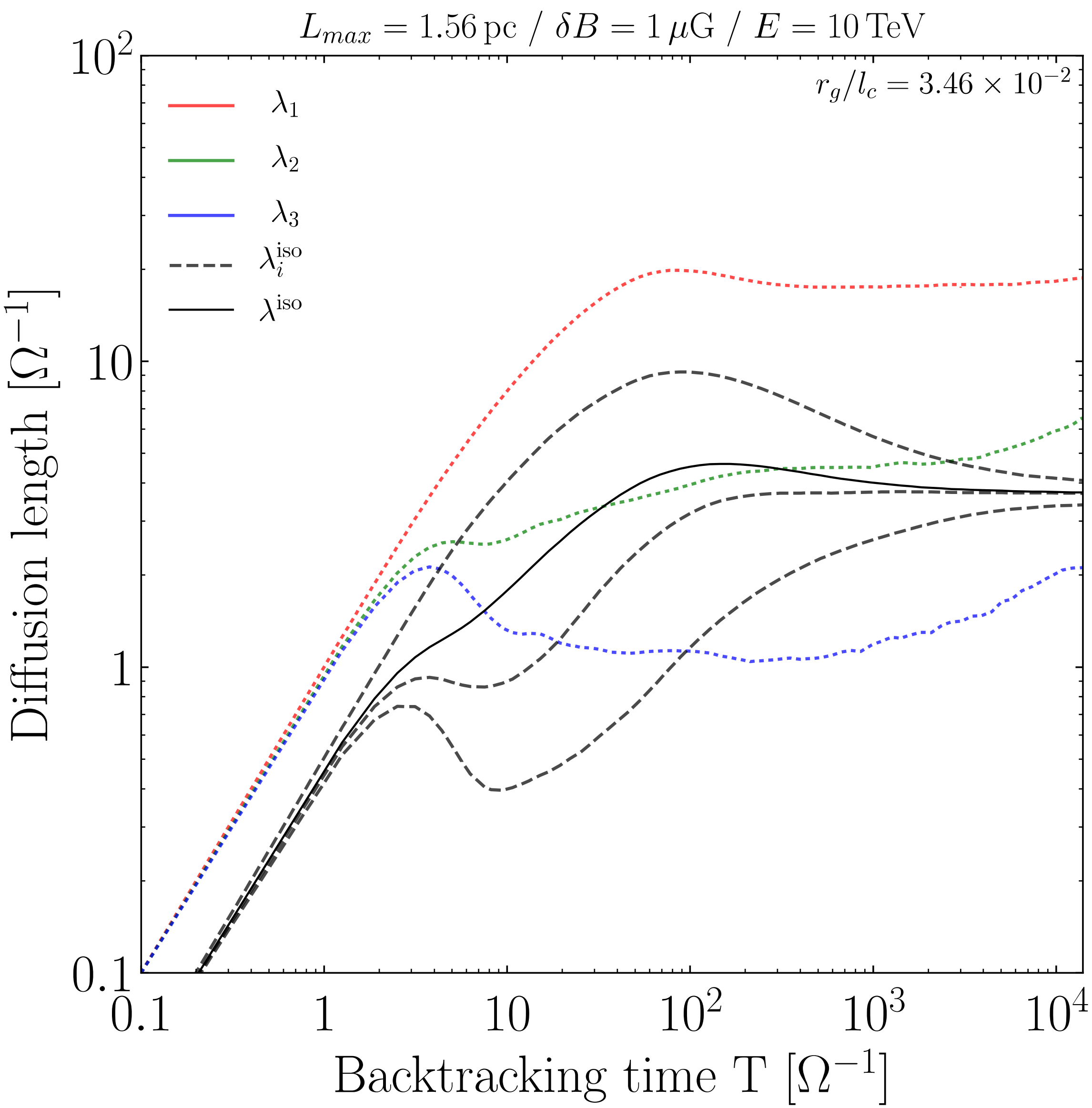

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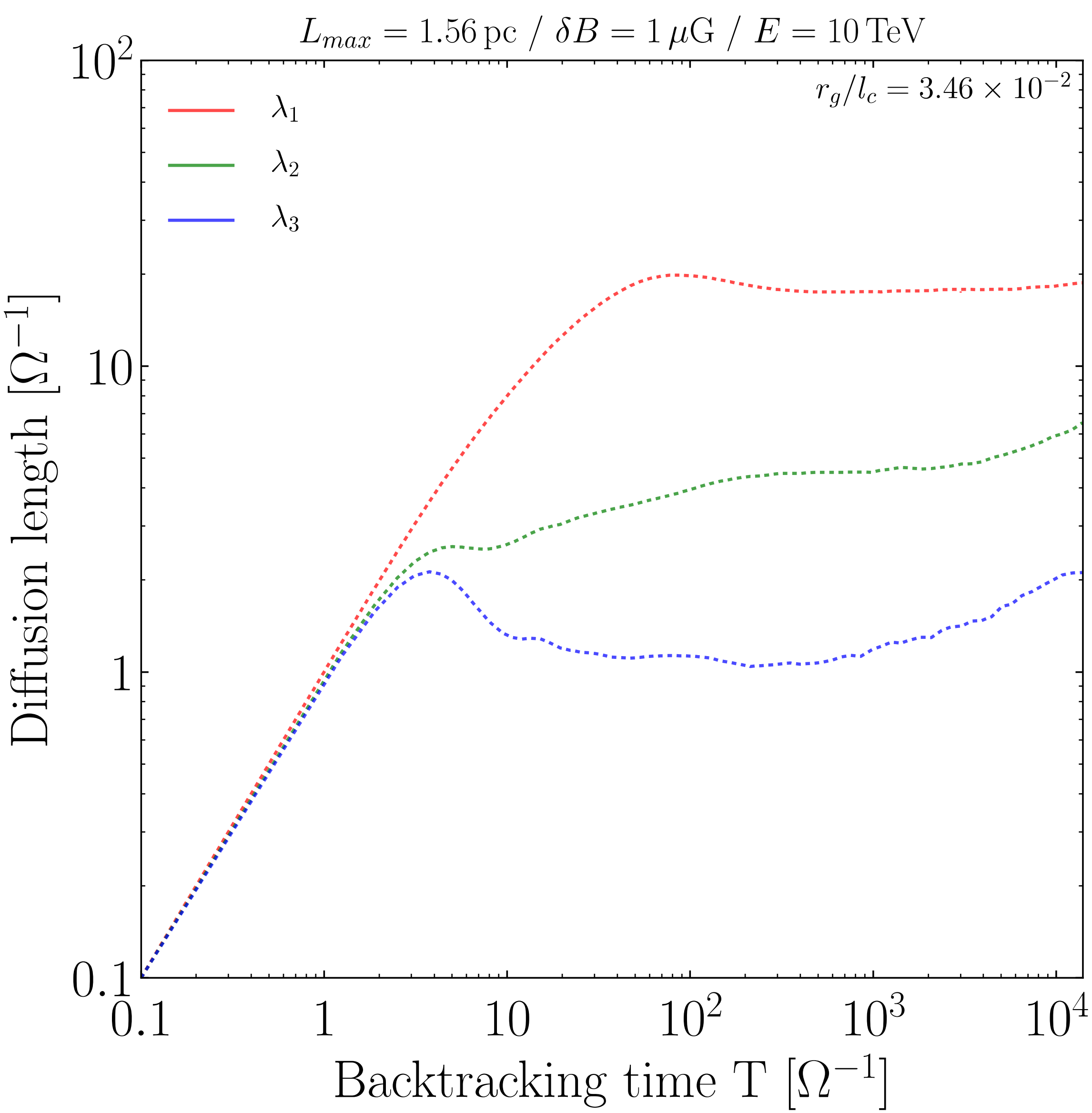

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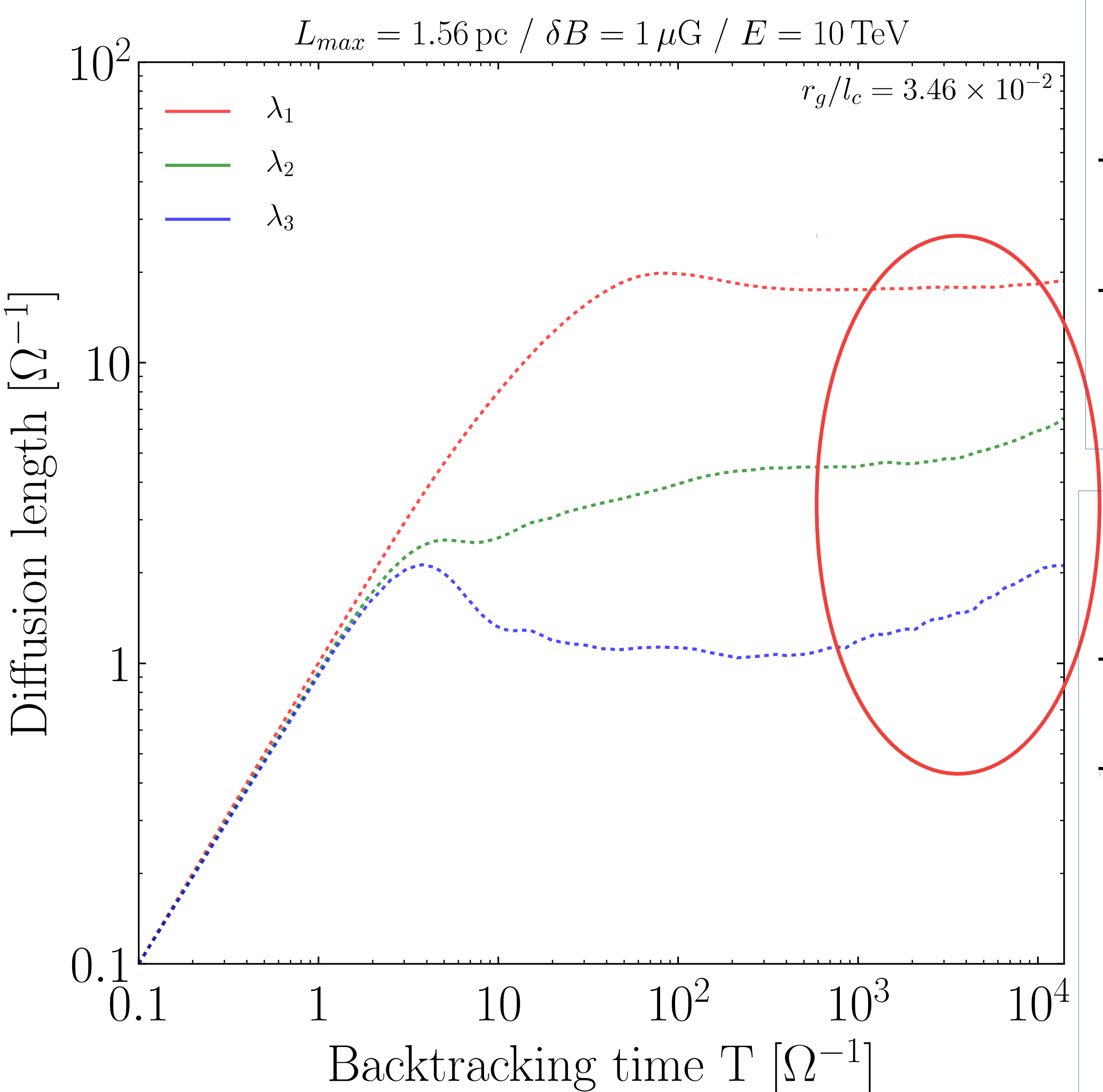

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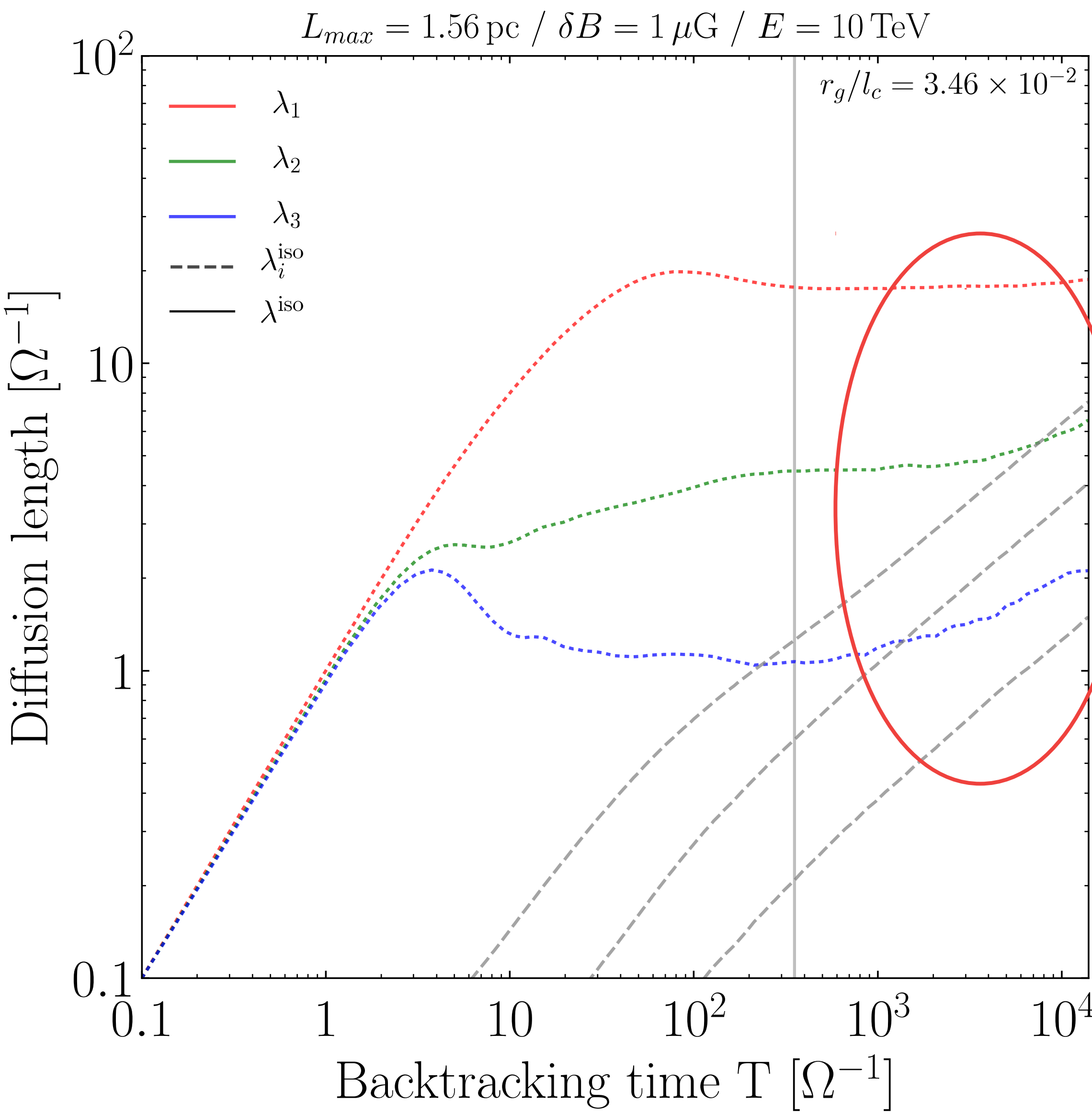

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→ Total noise well under control
But noise on each eigen value not known

→ Rescaling of the noise via a fit with constraints

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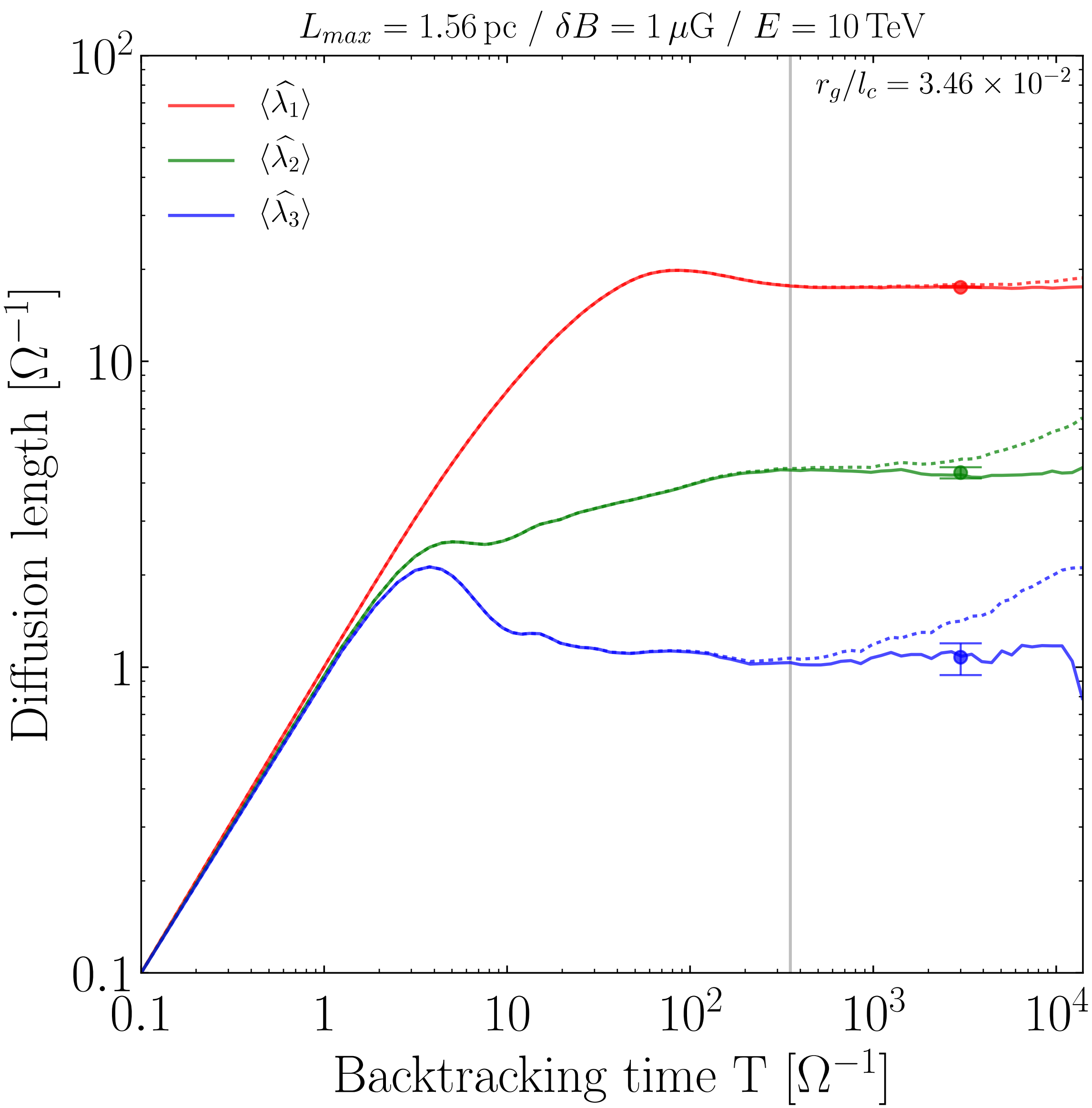

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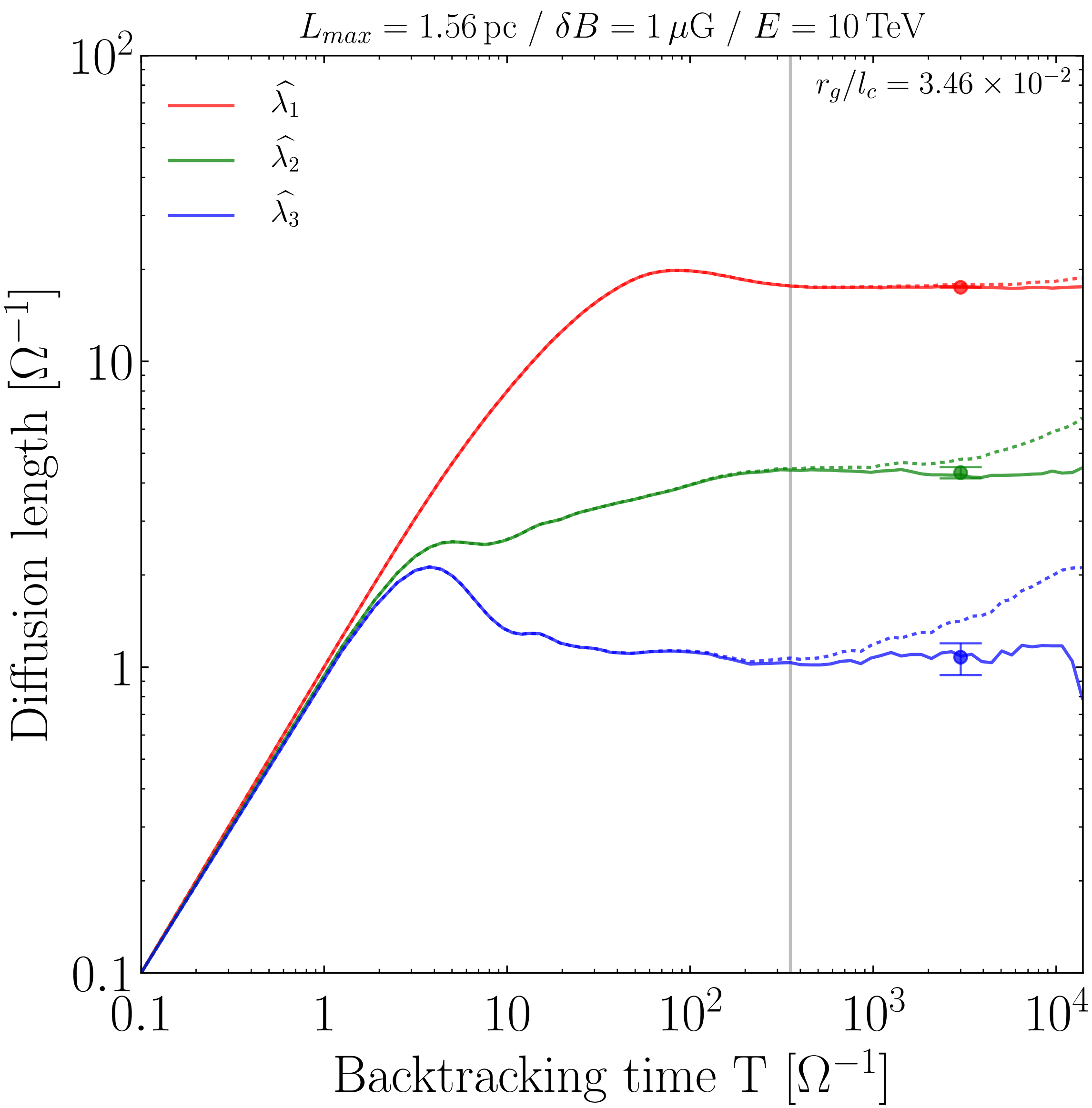


Local diffusion tensor strongly anisotropic

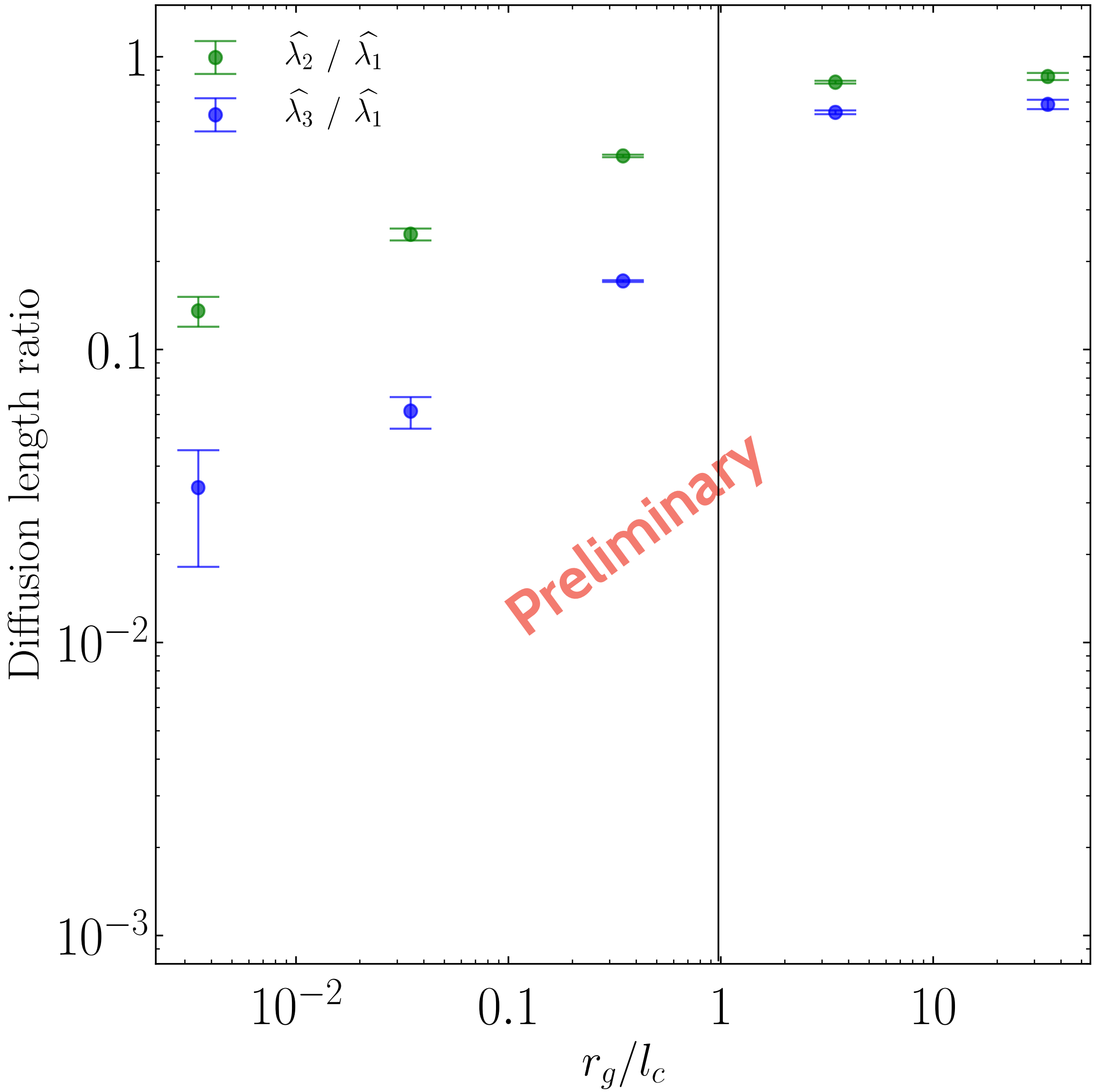
O(10) in isotropic tubulence for $r_g/l_c \sim 10^{-2}$

Main consequence

$$\frac{C_1}{4\pi} \simeq \mathrm{S}_{ij} \frac{\partial_i n \partial_j n}{n^2}$$

→ **Large projection effect** of cosmic-ray gardient

$$\frac{C_1}{4\pi} \propto \widehat{\lambda}_1^2 \frac{(\nabla_1 n)^2}{n^2} + \widehat{\lambda}_2^2 \frac{(\nabla_2 n)^2}{n^2} + \widehat{\lambda}_3^2 \frac{(\nabla_3 n)^2}{n^2}$$



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Evolution of the tensor with r_g/l_c

→ Increasing projection effect for small r_g/l_c

→ Convergence to isotropic diffusion $r_g/l_c > 1$

Conclusion & Prospect

CR dipole observations

→ Rapid phase flip and reduced dipole in the TeV-PeV range

Investigating local diffusion in isotropic turbulence

→ New methodology to study local diffusion (Nested grid & Backtracking)

→ In isotropic turbulence local diffusion is strongly anisotropic for $r_g/l_c < 1$

→ Evolution with particle rigidity towards isotropy for $r_g/l_c > 1$

Prospects

→ Challenges to remove the numerical noise for smaller r_g/l_c

→ Other magnetic configurations to probe

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