Fitting fractions of the X_{max} distributions at ultra high energies

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Simulations

- We test the ability of the method which uses 4 elements (p, He, N, Fe) to fit the observed X_{max} distributions.
- MC templates simulated with CONEX v4r37 for 8 primary species (p, He, C, N, O, Ne, Si and Fe) employing QGSJETII-04
- Adding the detector effects (acceptance and resolutions)
- Constructing large sample of X_{max} distributions with random concentrations of 8 elements and different statistics
- We found that a large abundance of Ne/Si (> 40%) will affect the reconstructed fractions of the elements considered into the fitting function →

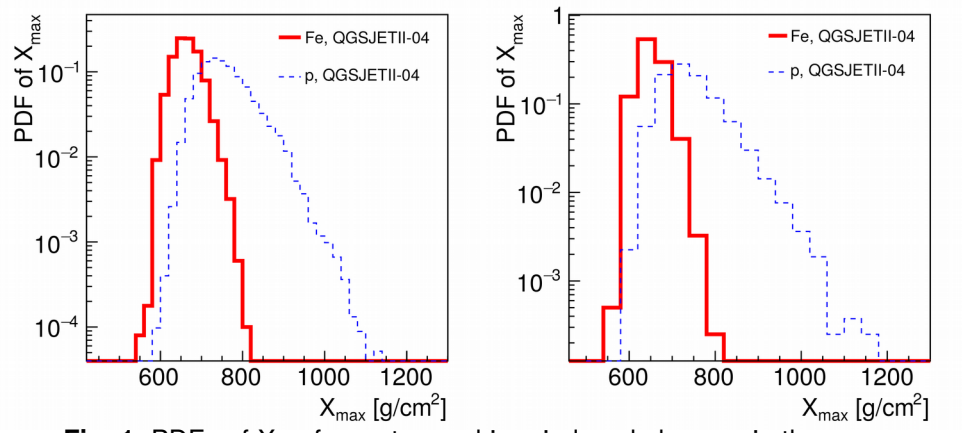

Fig. 1: PDFs of X_{max} for proton and iron induced showers in the energy range lg(E/eV) = [18.4 – 18.5] employing the QGSJETII-04 hadronic interaction model for Auger case (left) and TA case (right) .

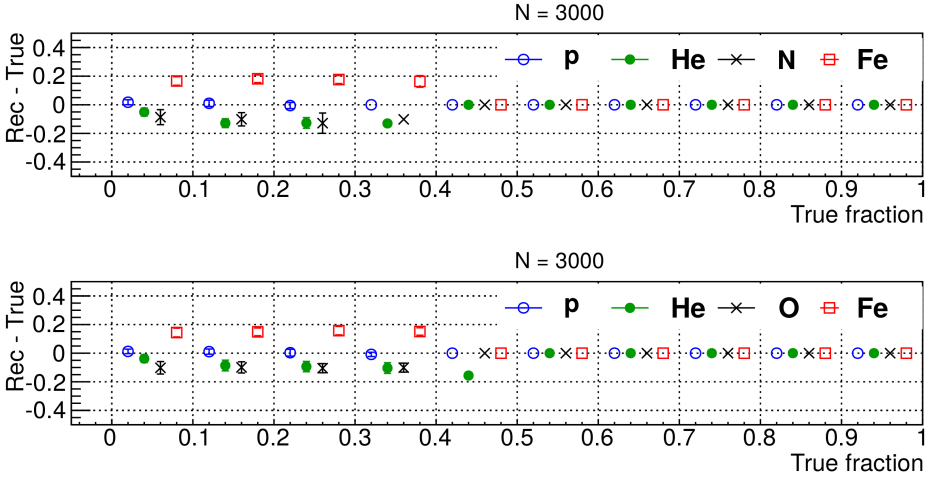

Fig. 2: The bias of the reconstructed fractions used in the fitting procedure as a function of their true prior fraction, when the concentration of Si is > 40% and the X_{max} distributions are fitted with (p, He, N, Fe) (up) and (p, He, O, Fe) (down), in the energy interval lg(E/eV) = [18.4 – 18.5]. The statistics in X_{max} distribution is N = 3000 events. The points corresponding to the true fraction interval [0.4 - 1] can be neglected.

Simulations

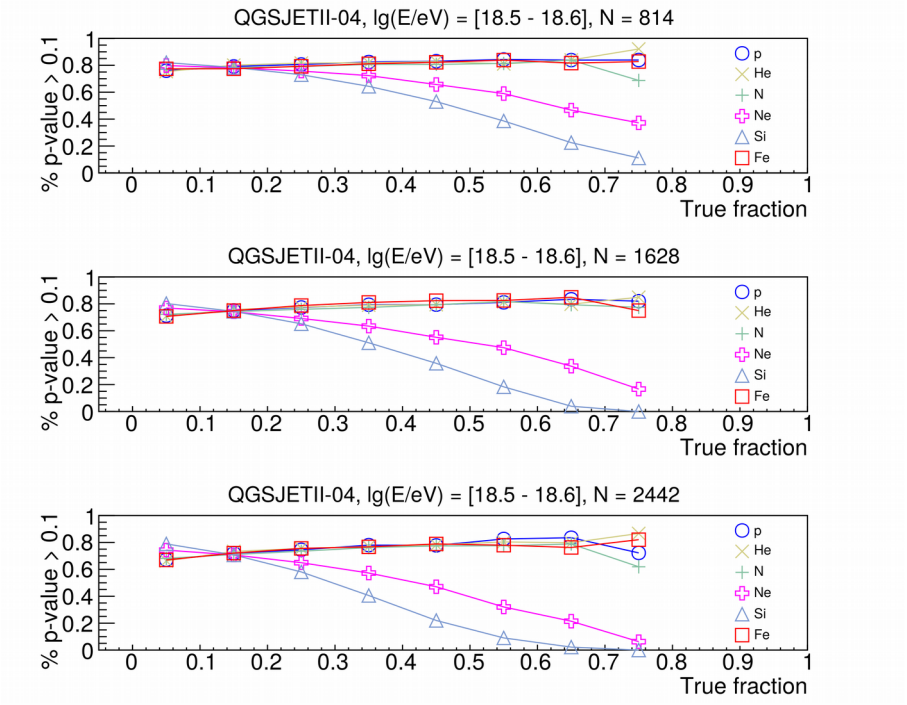

Fig. 3: Fraction of events with p-value > 0.1 as a function of prior abundances of different species corresponding to the energy interval lg(E/eV) = [18.5 – 18.6]. The fitting function includes only the four fixed elements (p, He, N and Fe). The statistics of X_{max} distributions is indicated on the top of the plots, corresponding to the Auger statistics $N = N_{Auger}$ *(up)*, $N = 2 \times N_{Auger}$ *(middle)* and $N = 3 \times N_{Auger}$ *(down)*.

In the fitting process we used the binned maximum-likelihood procedure, and the goodness-of-fit is characterized by the *p-value* parameter.

$$-\ln L = \sum_i y_i - n_i + n_i \ln(n_i/y_i)$$

$$p\text{-}value = 1 - \Gamma\left(\frac{ndf}{2}, \frac{\chi^2}{2}\right)$$

The probability of obtaining a good *p-value* decreases with the increase of abundances of Ne or Si and with increase of statistics in X_{max} distributions.

Fitting X_{max} distributions with all possible combinations of elements from a larger set (p, He, C, N, O, Ne, Si and Fe) and finding the "best combination" of elements which best describe the observed distribution

The "best combination" approach show that the experimental X_{max} distributions Auger (2014) and TA (2018) are well described with less than 4 elements.

The quality of the fit improves using this approach instead of fitting with the four fixed elements (p, He, N, Fe)

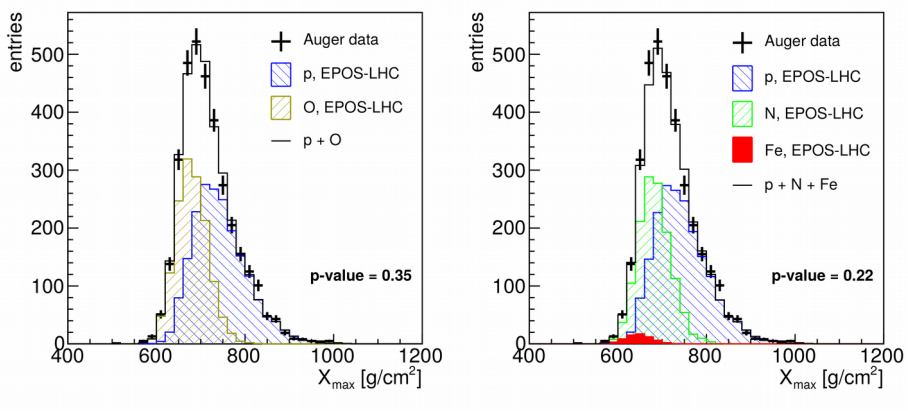

Fig. 4: X_{max} distribution recorded by Auger in the energy range lg(E/eV) = [17.9 – 18.0]. The reconstructed fractions using the "best combination" approach (*left*) and the method which uses the four elements (p. He, N and Fe) (*right*).

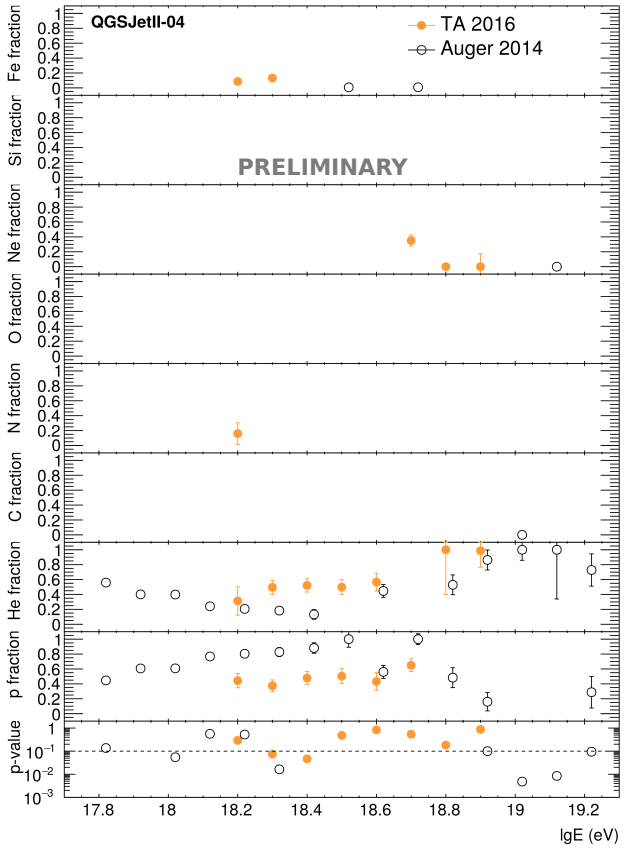

Fig. 5: Fitted fractions of individual nuclei in each energy interval obtained with the "best combination" approach predicted by QGSJETII-04 model.

Indirect comparison between Auger (2014) and TA (2016) X_{max} measurements following the "best combination" approach

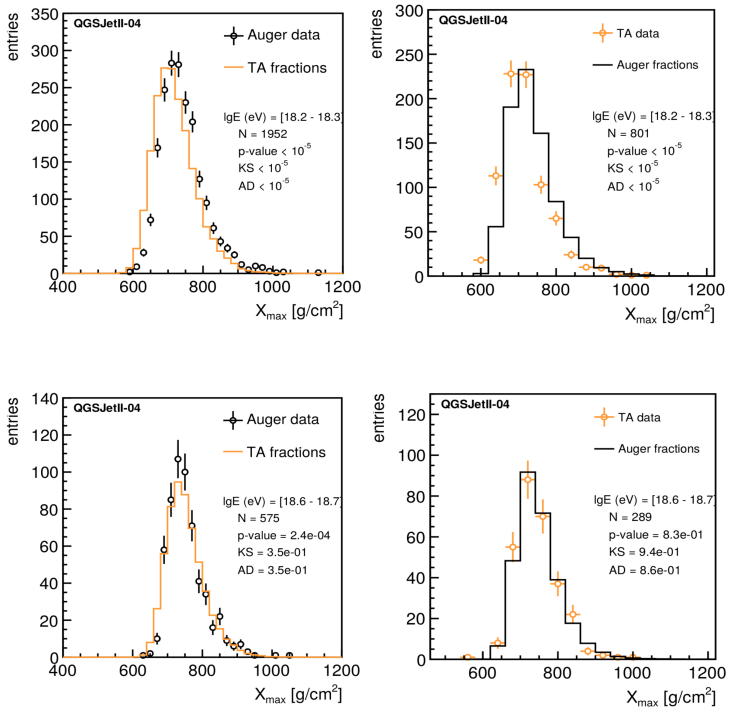

Fig. 6: Comparison between Auger data and PDFs of $X_{max}^{TA->Auger}$ (*left*) and TA data vs. PDFs of $X_{max}^{Auger->TA}$ (*right*) for the energy interval lg (*E*/eV) = [18.2 – 18.3] (*top*) and lg (*E*/eV) = [18.6 – 18.7] (*bottom*). The three parameters used to characterize the probability of compatibility are displayed on each plot.

	Auger vs. $X_{max}^{TA\to Auger}$			TA vs. $X_{max}^{Auger\to TA}$		
lgE (eV)	p-value	KS	AD	p-value	KS	AD
[18.2 - 18.3]	$< 10^{-5}$	$< 10^{-5}$	$< 10^{-5}$	$< 10^{-5}$	$< 10^{-5}$	$< 10^{-5}$
[18.3 - 18.4]	$< 10^{-5}$	$< 10^{-5}$	$< 10^{-5}$	$< 10^{-5}$	$< 10^{-5}$	$< 10^{-5}$
[18.4 - 18.5]	$< 10^{-5}$	2.1×10^{-4}	3.6×10^{-5}	$< 10^{-5}$	3.2×10^{-2}	4.3×10^{-3}
[18.5 - 18.6]	$< 10^{-5}$	1.1×10^{-2}	2.5×10^{-2}	4.0×10^{-5}	4.4×10^{-3}	2.0×10^{-3}
[18.6 - 18.7]	2.5×10^{-4}	3.5×10^{-1}	3.6×10^{-1}	8.3×10^{-1}	9.4×10^{-1}	8.6×10^{-1}
[18.7 - 18.8]	$< 10^{-5}$	6.1×10^{-5}	6.3×10^{-4}	4.4×10^{-5}	$< 10^{-5}$	5.7×10^{-4}
[18.8 - 18.9]	$< 10^{-5}$	$< 10^{-5}$	2.1×10^{-4}	7.9×10^{-2}	7.4×10^{-1}	3.7×10^{-1}
[18.9 - 19.0]	7.9×10^{-2}	1.6×10^{-2}	8.1×10^{-2}	9.0×10^{-1}	1.0	1.0

Table 1: The probability of compatibility between two data sets as computed by p-value, Kolmogorov-Smirnov and Anderson-Darling tests.

Conclusions

Fitting the X_{max} distributions with the same four elements on the entire energy spectrum, the reconstructed fractions of the individual nuclei will be biased in some energy intervals as a consequence of not including into the fitting function of some intermediate elements which are in fact present.

An appropriate method is to fit the observed distributions with all possible combination of elements from a larger set of primaries, finding in this way the "best combination" of elements to describe the data.

Applying this method to Auger(2014) and TA (2016) we found that the mass composition is dominated by protons and He nuclei (>70 %) on the entire energy spectrum, using predictions of QGSJETII-04 model.

An indirect comparison between the two data sets show a good degree of compatibility in some high energy bins, but worsening at lower energies.