



The asymptotic directions of approach and the magnetic rigidity cutoff of cosmic ray particles calculated for different airports.

[Marek Siłuszyk](#) (Siedlce University & Military University of Aviation)

[Krzysztof Iskra](#) (Military University of Aviation)

[Witold Woznak](#) (Polish Gas Company)

[Michał Borkowski](#) (Military University of Aviation)

[Tomasz Zienkiewicz](#) (Military University of Aviation)

Abstract:

The calculation of asymptotic directions of approach of cosmic ray particles is an important tool in the determination of the rigidity cutoff for a given geographical site. We present the results of computations of the asymptotic latitude and asymptotic longitude and the magnetic rigidity cutoff for the airports (Apatity, Oulu, Warsaw, Lae, Buenos Aires, Wellington and Mc Murdo) located at different latitudes and longitudes based on the numerical integration of equations of motion of charged particles of cosmic radiation in the Earth's magnetic field. The initial distance from the center of the Earth was taken as 20 km above the surface. At about this altitude, most cosmic rays undergo nuclear collisions. Calculations were made for the model of the International Geomagnetic Reference Field (IGRF) in 2015.

The equation of motion of the particle in the Gaussian system of units is:

$$m \frac{d^2 \vec{r}}{dt^2} = \frac{e}{c} (\vec{v} \times \vec{B}) \quad (1)$$

and this may be written in terms of the spherical coordinate system as follows

$$\left\{ \begin{array}{l} \frac{dv_r}{dt} = \frac{e}{mc} (v_\theta B_\varphi - v_\varphi B_\theta) + \frac{v_\theta^2}{r} + \frac{v_\varphi^2}{r} \\ \frac{dv_\theta}{dt} = \frac{e}{mc} (v_\varphi B_r - v_r B_\varphi) - \frac{v_r v_\varphi}{r} + \frac{v_\varphi^2}{r \operatorname{tg} \theta} \\ \frac{dv_\varphi}{dt} = \frac{e}{mc} (v_r B_\theta - v_\theta B_r) - \frac{v_r v_\varphi}{r} - \frac{v_\varphi v_\theta}{r \operatorname{tg} \theta} \end{array} \right. \quad (2)$$

$$v_r = \frac{dr}{dt}$$

$$v_\theta = r \frac{d\theta}{dt} \quad (3)$$

$$v_\varphi = r \sin \theta \frac{d\varphi}{dt}$$

$$\left\{ \begin{array}{l} \frac{dt_r}{ds} = \frac{e}{vmc} (t_\theta B_\varphi - t_\varphi B_\theta) + \frac{t_\theta^2}{r} + \frac{t_\varphi^2}{r} \\ \frac{dt_\theta}{ds} = \frac{e}{vmc} (t_\varphi B_r - t_r B_\varphi) - \frac{t_r t_\theta}{r} + \frac{t_\varphi^2}{r \operatorname{tg} \theta} \\ \frac{dt_\varphi}{ds} = \frac{e}{vmc} (t_r B_\theta - t_\theta B_r) - \frac{t_r t_\varphi}{r} - \frac{t_\varphi t_\theta}{r \operatorname{tg} \theta} \end{array} \right. \quad (5)$$

$$U = r_z \sum_{n=1}^{\infty} \left(\frac{r_z}{r}\right)^{n+1} \sum_{m=0}^n (g_n^m \cos m \varphi + h_n^m \sin m \varphi) P_n^m(\cos \theta) \quad (6)$$

$$\left\{ \begin{array}{l} B_\theta = -\frac{1}{r} \frac{dU}{d\theta} = -\sum_{n=1}^{\infty} \left(\frac{r_z}{r}\right)^{n+2} \sum_{m=0}^n (g_n^m \cos m \varphi + h_n^m \sin m \varphi) \frac{dP_n^m(\cos \theta)}{d\omega} \\ B_\varphi = -\frac{1}{r \sin \theta} \frac{dU}{d\varphi} = -\sum_{n=1}^{\infty} \left(\frac{r_z}{r}\right)^{n+2} \sum_{m=0}^n (m g_n^m \sin m \varphi + m h_n^m \cos m \varphi) \frac{P_n^m(\cos \theta)}{\sin \theta} \\ B_r = -\frac{dU}{dr} = -\sum_{n=1}^{\infty} \left(\frac{r_z}{r}\right)^{n+2} (n+1) \sum_{m=0}^n (g_n^m \cos m \varphi + h_n^m \sin m \varphi) P_n^m(\cos \theta) \end{array} \right. \quad (7)$$

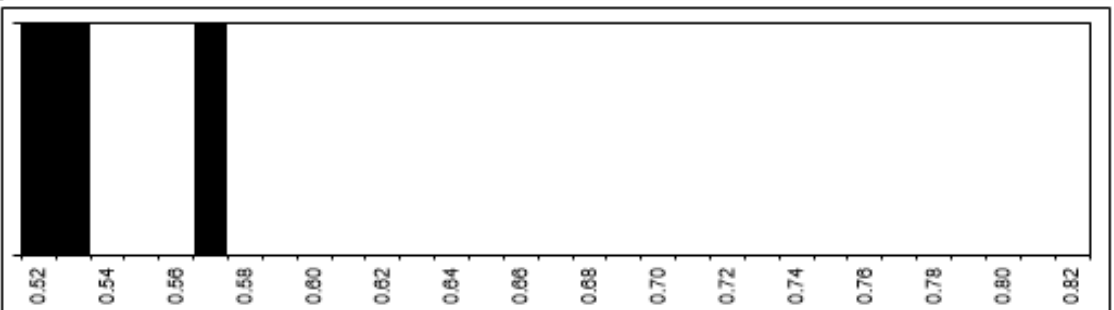
4 Table 1. Localization parameters of selected eight airports

No	Airport	Geographic latitude [degrees]	Geographic longitude[degrees]	Height [m]
1	Oulu	64,91	25,37	14
2	Apatity	67,45	33,58	160
3	Warsaw	52,15	20,96	110
5	Lae	8,92	166,25	1
6	Buenos Aires	-34,82	-58,53	20
7	Wellington	-41,32	174,8	13
8	Mc Murdo	-71,85	166,47	10

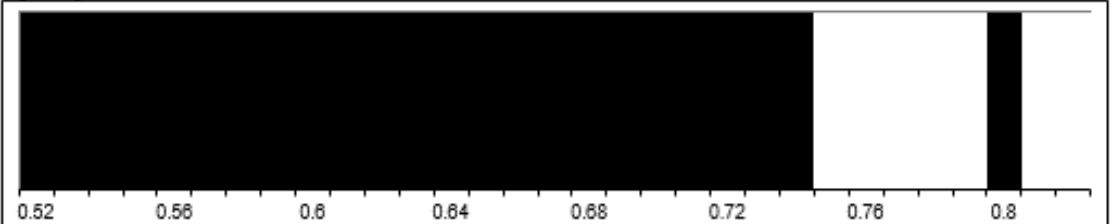
Table 2. Cut-off rigidities and the width of the penumbra zone for vertical components for seven airports

No	Airports	Vertical penumbral zone			
		R _{st} [GV]	R _{ref} [GV]	R _m [GV]	Width of penumbral zone in GV
1	Apatity	0,53	0,54	0,58	0,05
2	Oulu	0,74	0,75	0,81	0,07
3	Warsaw	2,61	2,85	3,13	0,52
5	Lae	15,34	15,34	15,35	0,01
6	Buenos Aires	7,54	8,17	8,84	1,3
7	Wellington	2,91	3,27	3,41	0,5
8	Mc Murdo	0,09	0,1	0,12	0,03

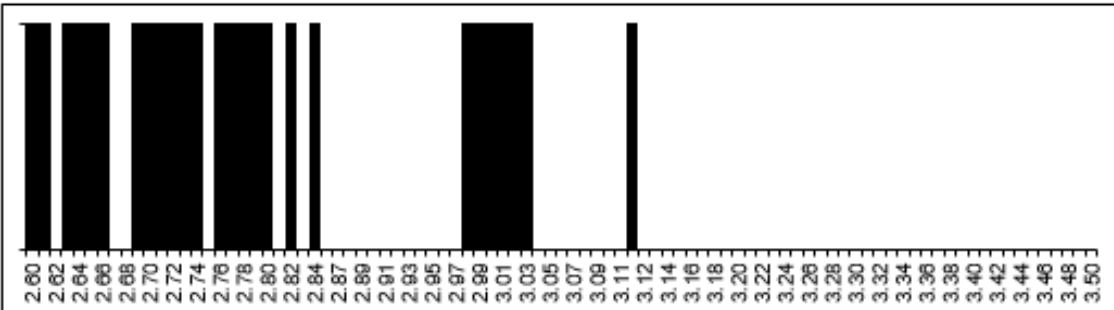
	Inclined components			Width of penumbral zone in GV
	R _{st} [GV]	R _{ref} [GV]	R _m [GV]	
Apatity				
Azimuth angle		Zenith angle	16	
65	0,53	0,54	0,57	0,04
155	0,53	0,54	0,56	0,03
245	0,52	0,53	0,56	0,04
355	0,51	0,52	0,54	0,03
Azimuth angle		Zenith angle	32	
65	0,53	0,54	0,56	0,03
155	0,53	0,54	0,56	0,03
245	0,52	0,53	0,57	0,05
355	0,52	0,53	0,57	0,05



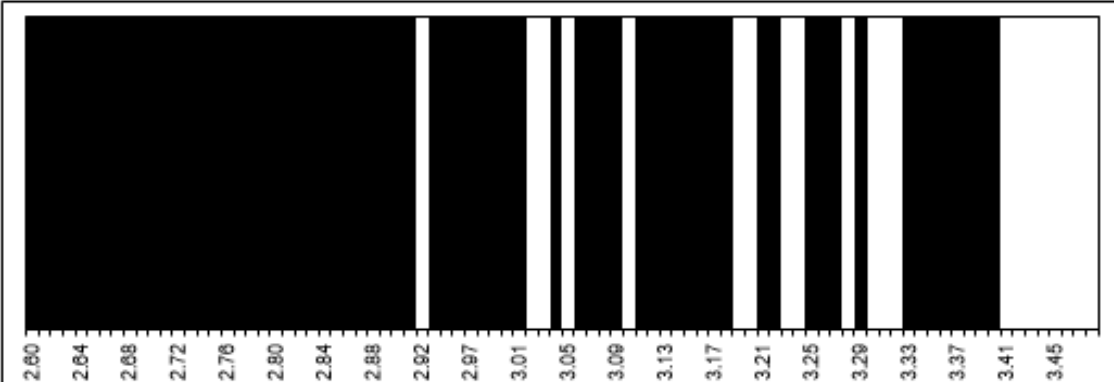
Apatity



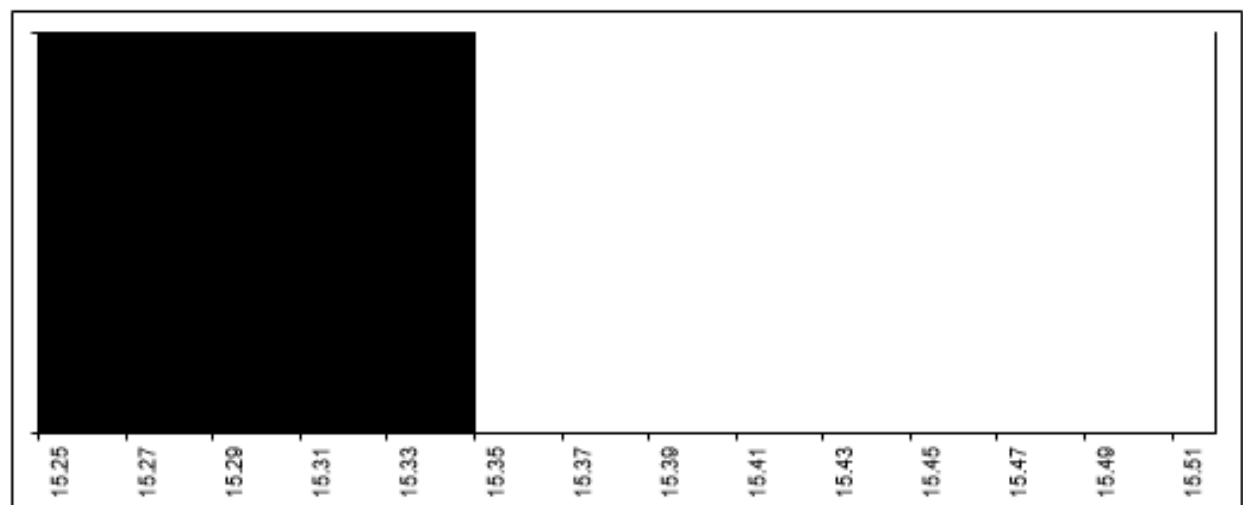
Oulu



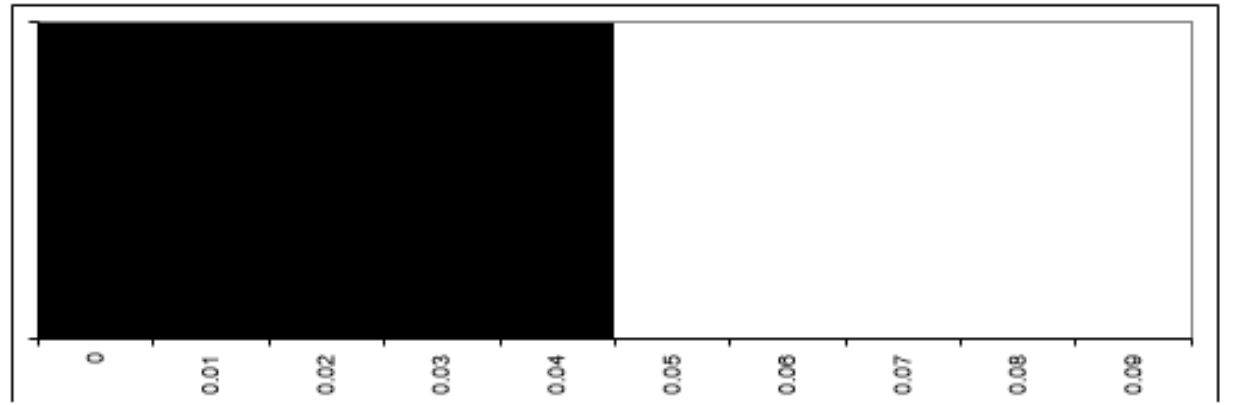
Warsaw



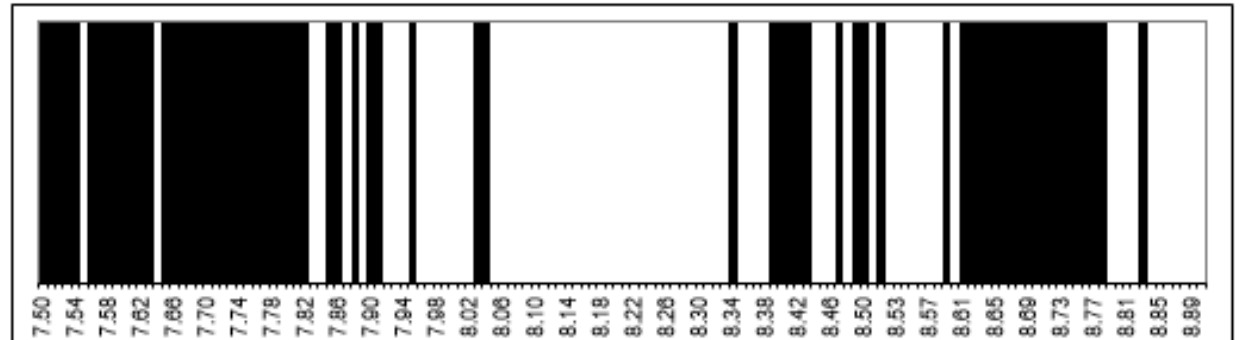
Wellington



Lae



Mc Murdo



Buenos Aires

Conclusions

1. Asymptotic directions (asymptotic latitude and longitude) of the arrival of charged particles and magnetic cutoff rigidity were determined for the airports: Apatity, Oulu, Warsaw, Lae, Buenos Aires, Wellington and Mc Murdo.
2. The magnetic cut-off rigidity depends on the latitude. With the increase of latitude, the magnetic stiffness decreases, i.e. cosmic ray particles reach the Earth more easily from the poles than the equator
3. For the airports located at medium latitude, we observe a penumbra zone, that contains a family of allowed and forbidden trajectories of cosmic ray particles and magnetic cutoff rigidity is expressed by R_{ef} (Warsaw, Buenos Aires, Wellington)
4. For the airports located at high latitude, (Apatity, Mc Murdo and Oulu) the penumbra does not exist or is very narrow and magnetic cut-off rigidity is expressed by R_{st} in the absence of penumbra or by R_{ef} if penumbra exists.
5. In low latitudes (Lae airport), the penumbra does not appear and magnetic cutoff rigidity is expressed by R_m
6. Analysis of the magnetic cut-off rigidity for the inclined components, we observe east-west asymmetry.
7. Knowledge of asymptotic directions and the magnetic cutoff rigidity, it is important from the point of view of the flight safety of both passenger and military aircraft and study of the different classes of the cosmic rays variations intensity and anisotropy.

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Thank You



Marek Siluszyk
msiluszyk(at)uph.edu.pl