Stochastic Acceleration of UHECRs in the early afterglows of gamma-ray bursts:

concurrence of jet's dynamics and wave-particle interactions

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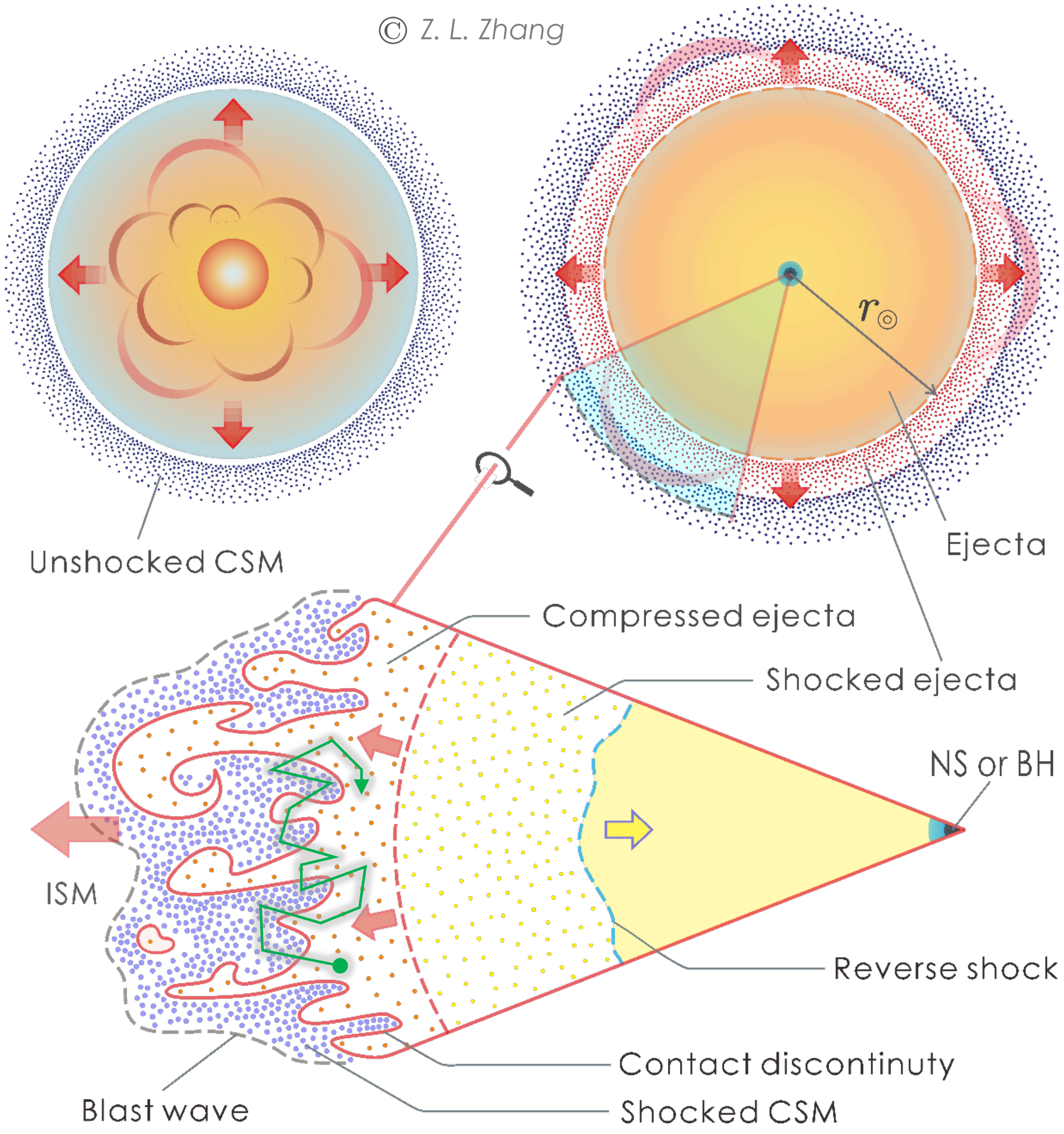
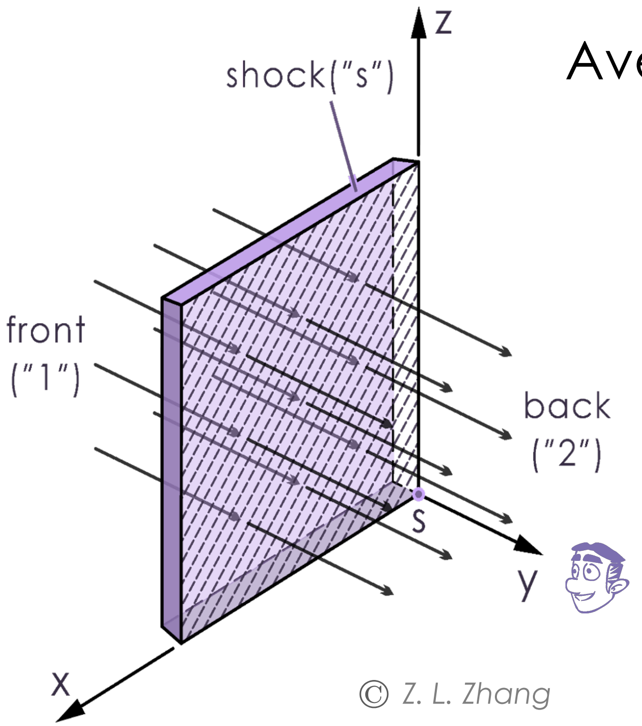
Acceleration mechanisms



Shock acceleration (1st Fermi)



Average energy gain/cycle

$$\left\langle \frac{\Delta E}{E} \right\rangle \sim \frac{v}{c}$$

Symmetry (1 and 2)
"head-on" collisions

Asymmetry (1 and 2)
advect and escape

Bell MNRAS 1978

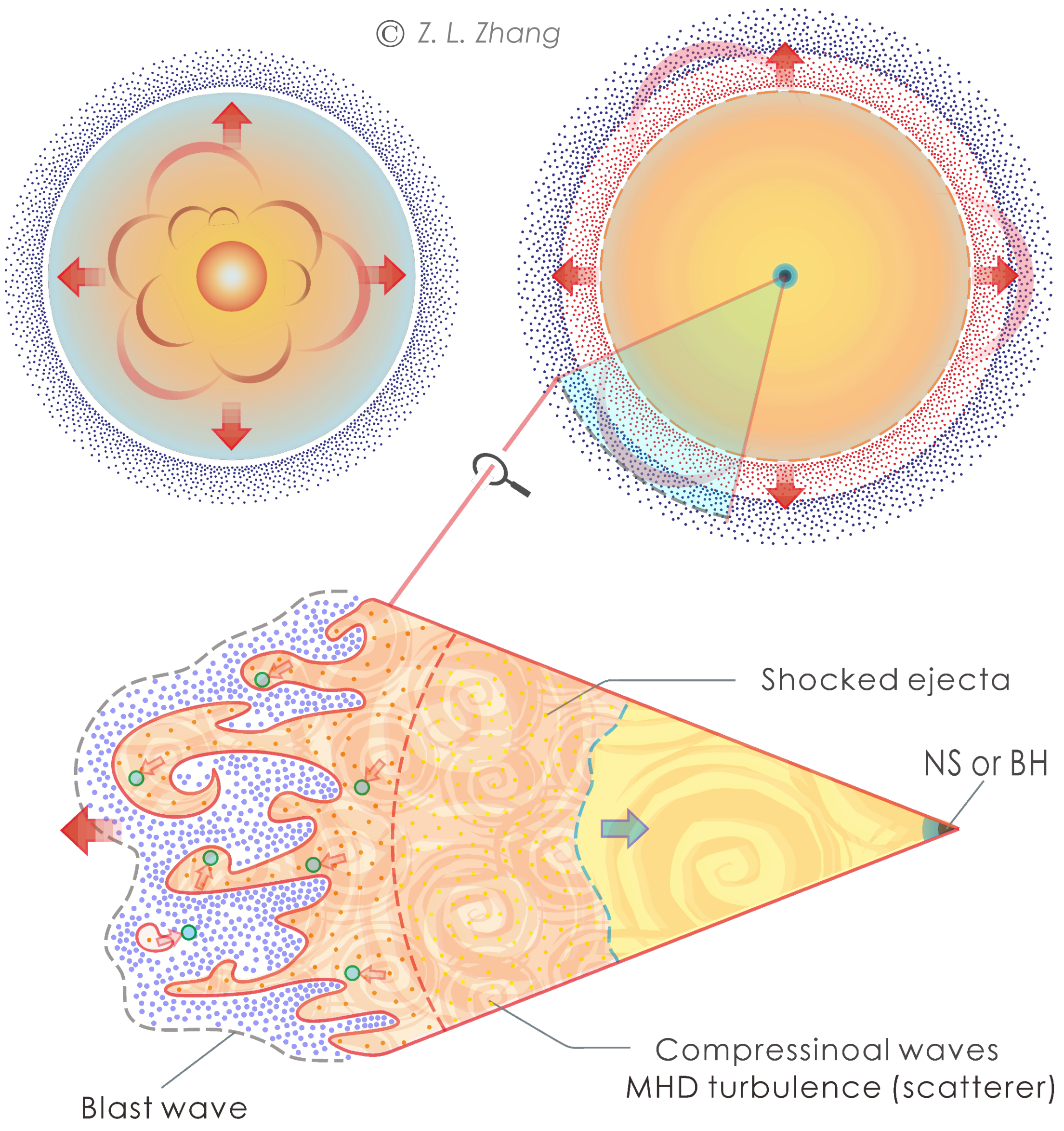
Non-relativistic shock: more effective i.e. DSA

Gallant & Achterberg MNRAS 1999

Relativistic shock: less effective

$\Gamma_s^2 \to 2$ after the 1st crossing circle

Acceleration mechanisms



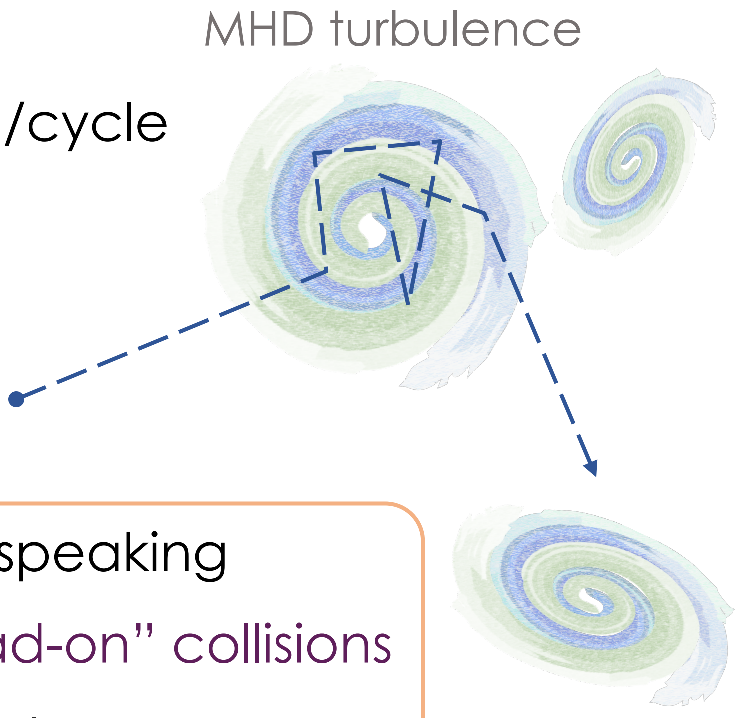
Stochastic acceleration (2nd Fermi)

Fermi PR 1949 & ApJ 1954

Average energy gain /cycle

$$\left\langle \frac{\Delta E}{E} \right\rangle \sim \left(\frac{v}{c} \right)^2$$

Low efficiency !



Statistically speaking

probability of "head-on" collisions

Is larger than

probability of "tail-on" collisions

It naturally produces a power-law spectrum of accelerated relativistic particles.

Acceleration mechanisms within GRBs

Katz *et al.* JCAP 2009
Aartsen *et al.* ApJ 2017
Gallant & Achterberg MNRAS 1999
Lemoine & Pelletier ApJ 2003
Alves Batista *et al.* JCAP 2019

1. Baryon loading factor (prompt emission)

CRs $\sim 10^{44}\,\mathrm{erg\ Mpc^{-3}yr^{-1}}$ γ-ray $\sim 10^{43}\,\mathrm{erg\ Mpc^{-3}yr^{-1}}$

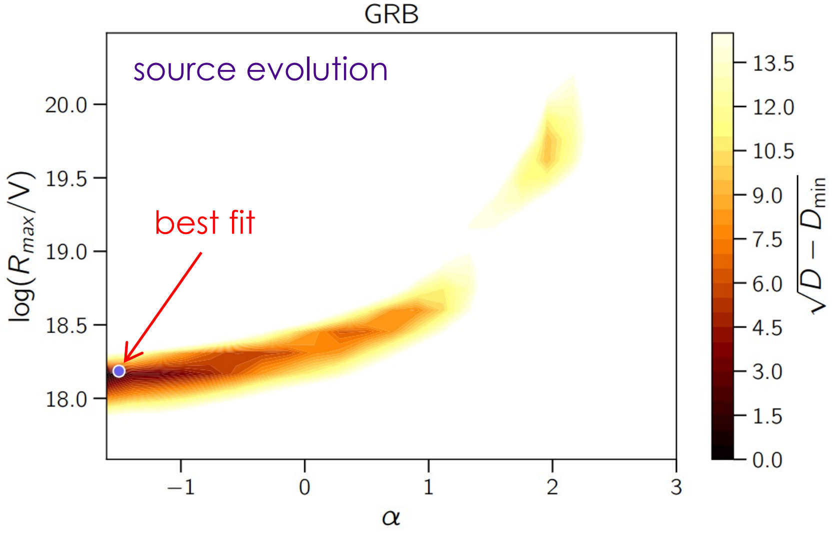
local GRB rate $\sim 1\,\mathrm{Gpc^{-3}yr^{-1}}$ $L_\gamma \sim 10^{52}\,\mathrm{erg\ s^{-1}}$

For relativistic shock acceleration, $p \equiv \alpha \gtrsim 2$

Only $\sim 10\%$ of total CR energy beyond ankle $\varepsilon > 10^{18.5}\,\mathrm{eV}$

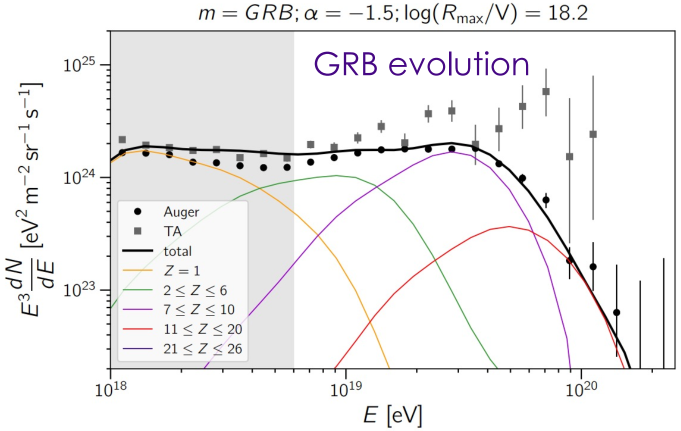
$\Rightarrow$ It require a baryon loading factor : $\eta \equiv \dfrac{E_{\mathrm{tot}}(\mathrm{CRs})}{E_\gamma(\mathrm{CRs} \rightsquigarrow \gamma)} \sim 100$

Intension with IceCube's result ! ~ 10



2. Acceleration rate (afterglow)

For ultra-relativistic shock, $\Gamma_s^2 \to 2$ after the 1st crossing circle

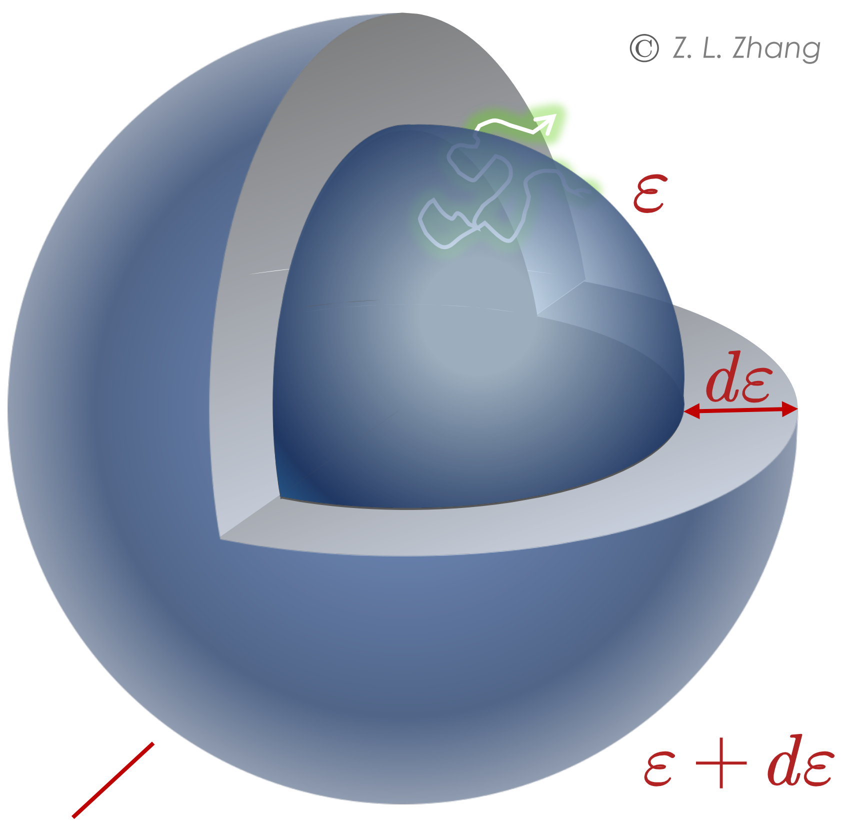


3. Hard-spectrum problem

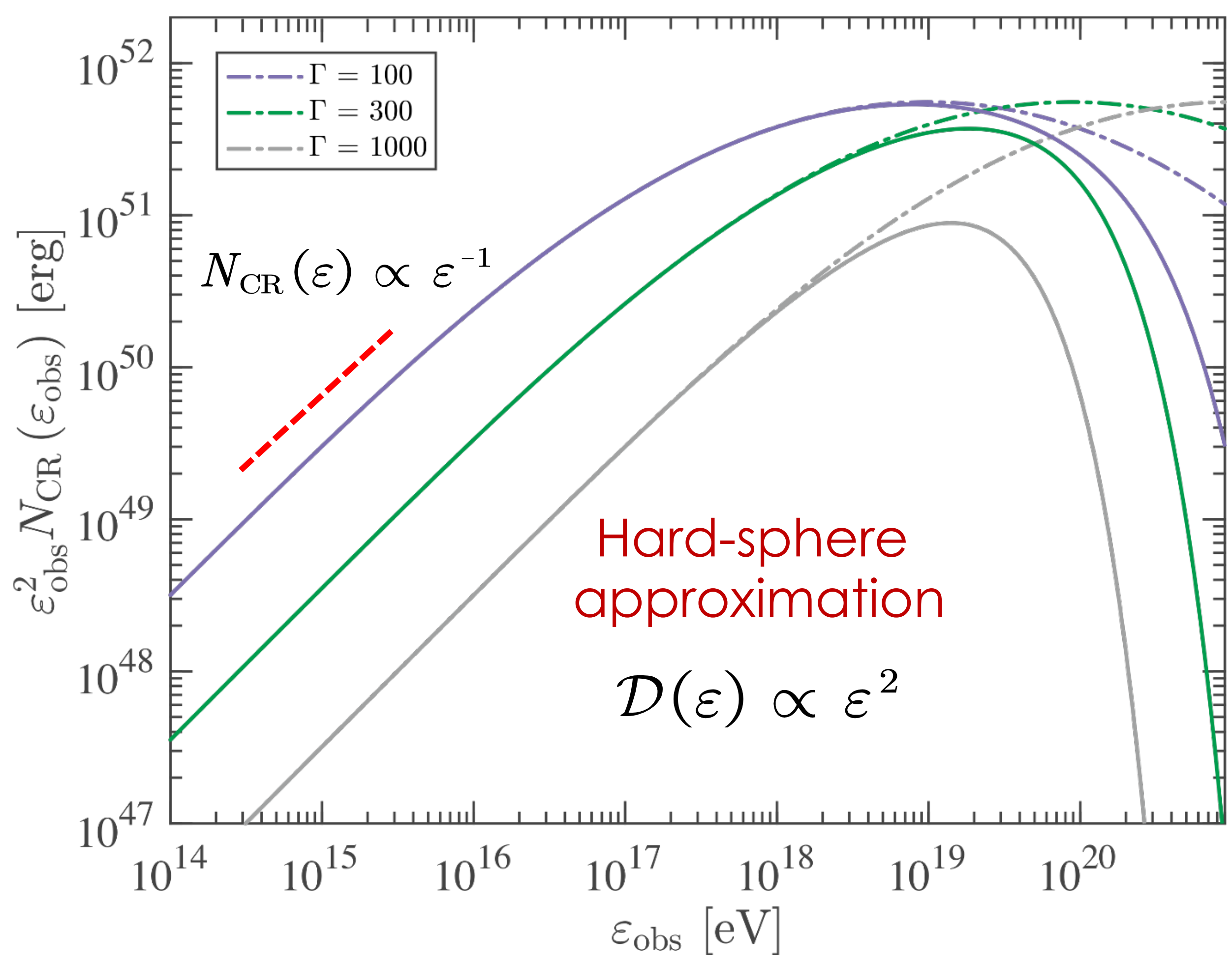
The results of the Auger fit indicate a hard injection spectrum with $p \lesssim 1$

Acceleration mechanisms within GRBs

Diffusion in energy space



Asano & Mészáros PRD 2016



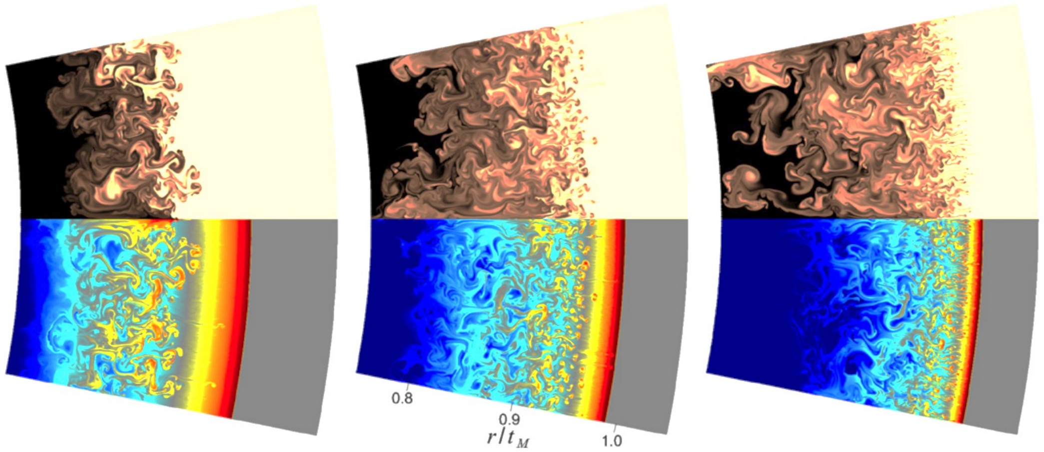
Larger volume means higher acceleration!

$$\frac{\partial N(\varepsilon,t)}{\partial t} = \frac{\partial}{\partial \varepsilon}\left[\mathcal{D}(\varepsilon)\frac{\partial N(\varepsilon,t)}{\partial \varepsilon}\right] - \frac{\partial}{\partial \varepsilon}\left[\frac{2\mathcal{D}(\varepsilon)}{\varepsilon}N(\varepsilon,t)\right] + \dot{N}_{\rm inj}(\varepsilon,t)$$

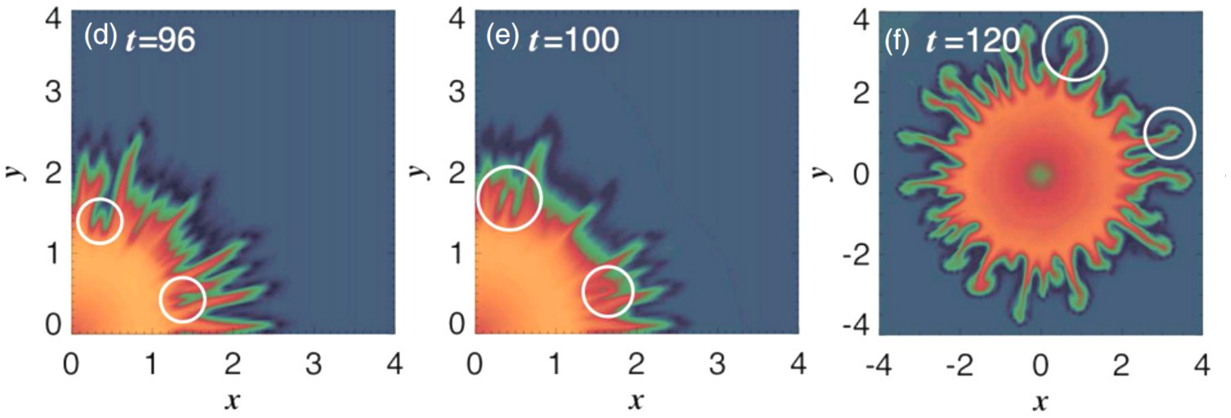
Excitation of turbulence

Instabilities and turbulence injection

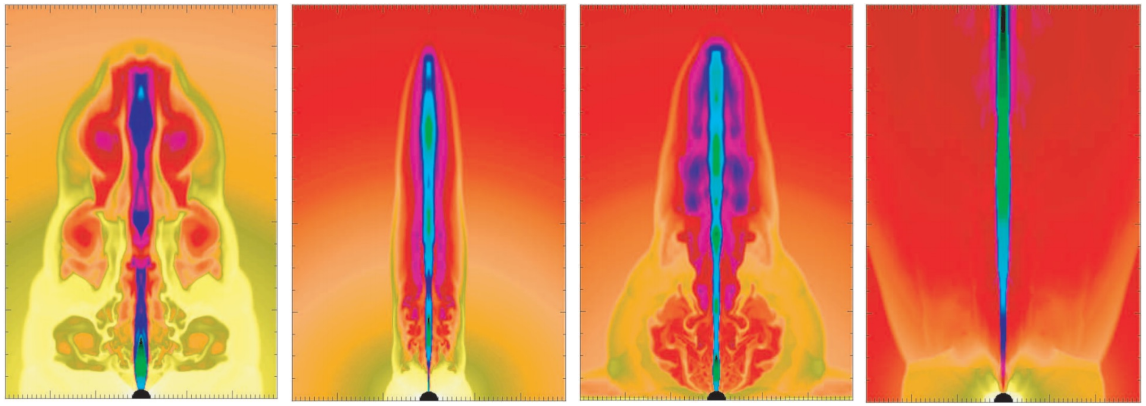
Rayleigh-Taylor (Duffell *et al.* ApJ 2013)



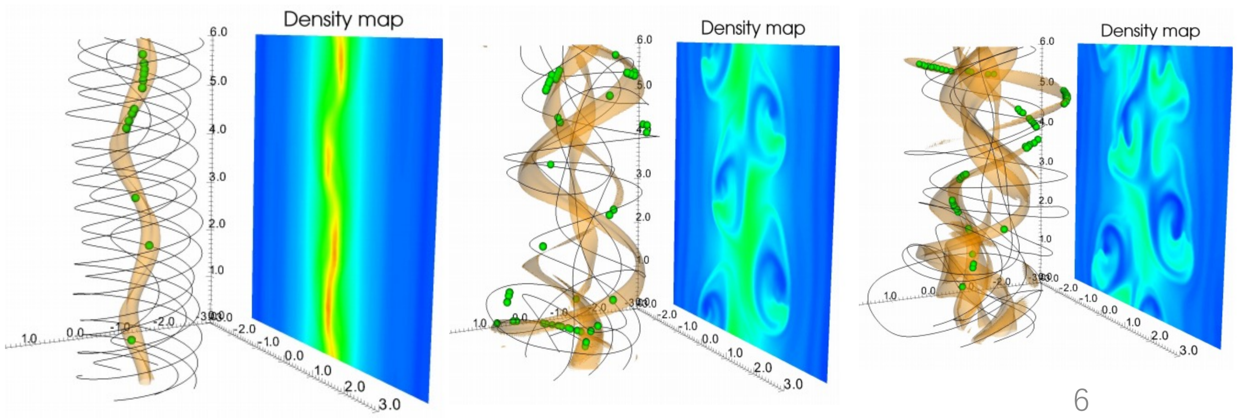
Richtmyer-Meshkov (Matsumoto *et al.* ApJ 2013)



Kelvin-Helmholtz (Zhang *et al.* ApJ 2003)



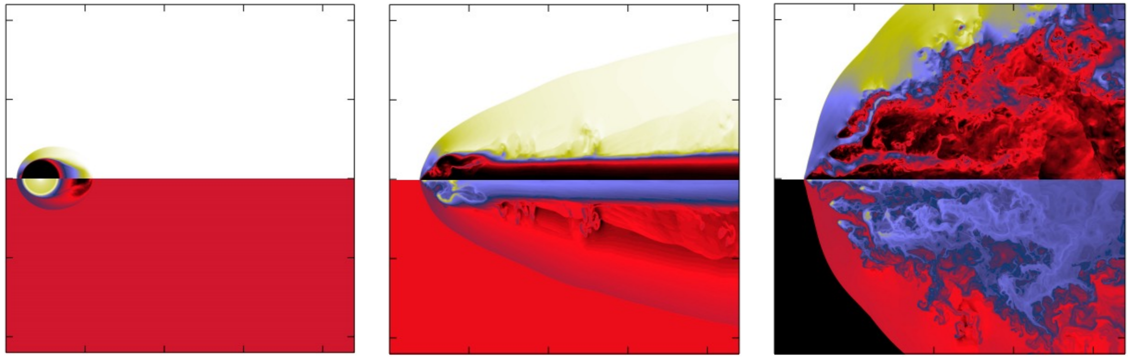
Kink instability (Medina-Torrejón *et al.* ApJ 2021)



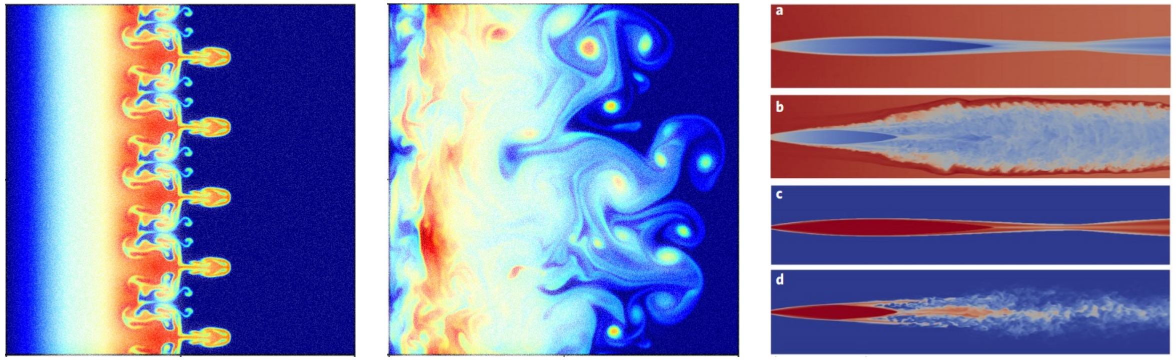
Excitation of turbulence

Instabilities and turbulence injection

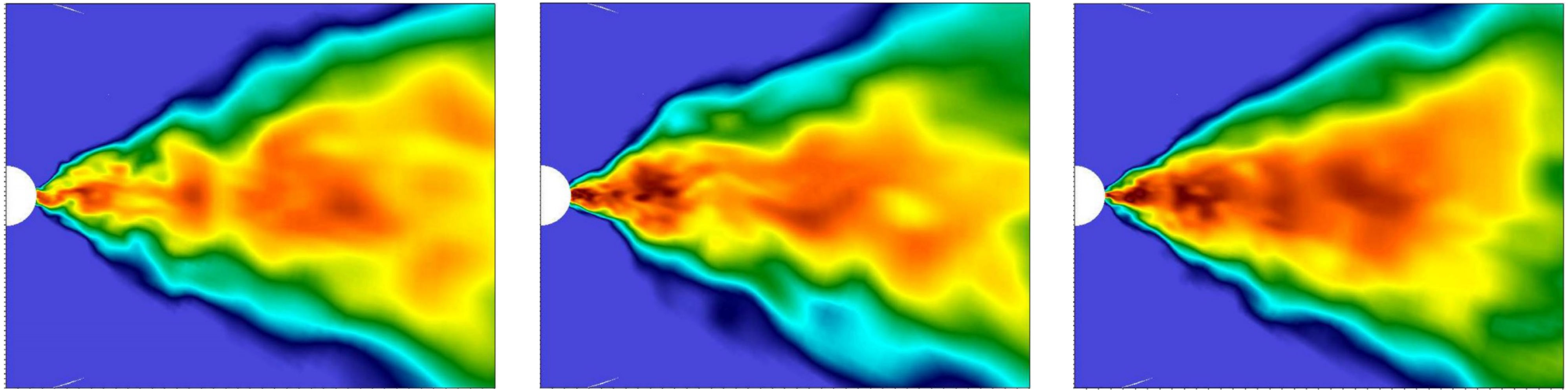
Star-jet interaction (Perucho *et al.* A&A 2017)



Centrifugal instability (Gourgouliatos *et al.* MNRAS, NA 2018)

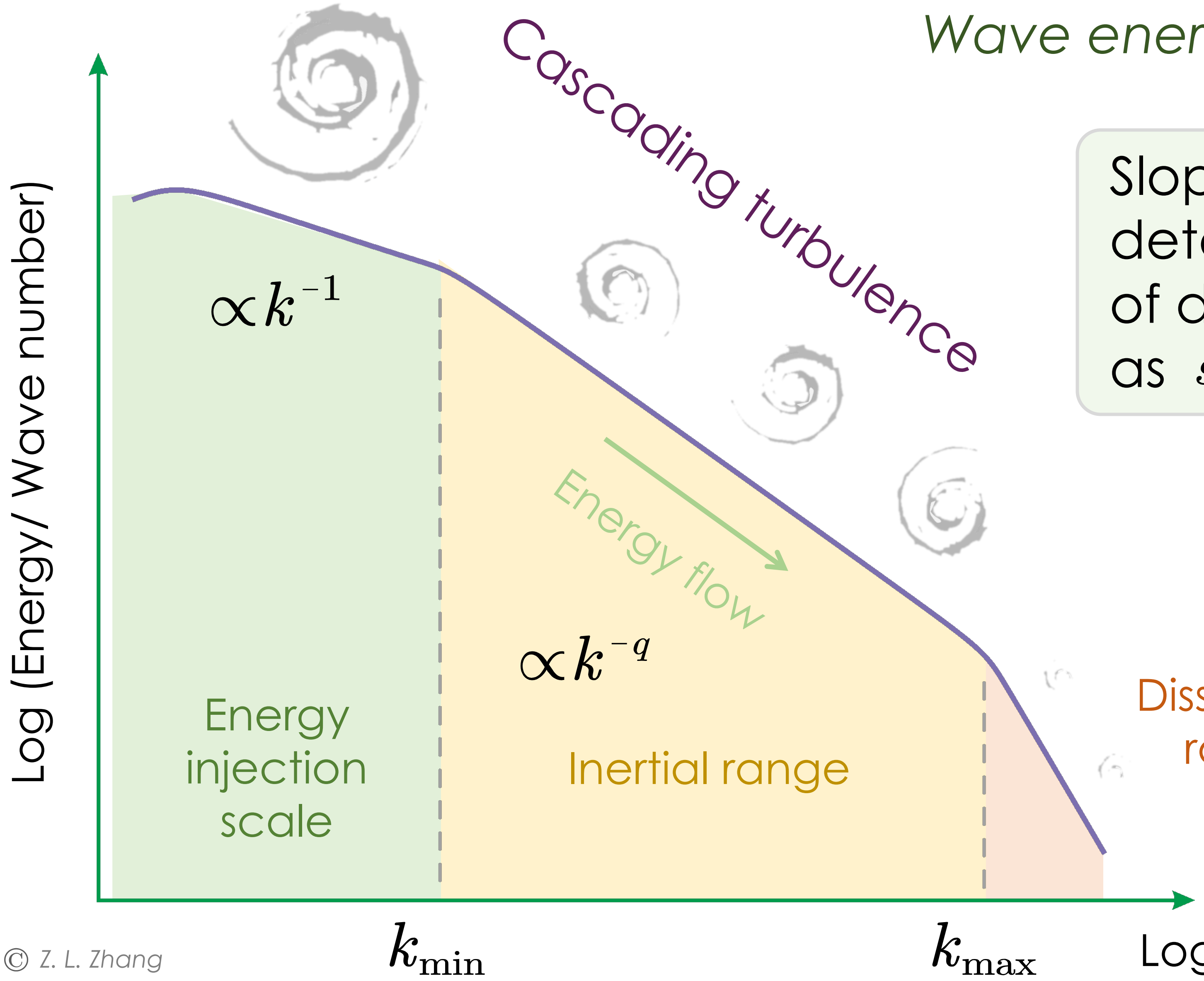


Magnetorotational instability (Pimentel Diaz *et al.* MNRAS 2021)



Injection of turbulence

Wave energy distribution: $W(k) \propto k^{-q}$



Slope of power spectrum $\propto k^{-q}$ determines energy dependence of diffusion coefficient $\mathcal{D}(E) \propto E^{s}$ as $s = 2 - q$.

$$q = \begin{cases} 1: \text{Bohm} \\ \frac{3}{2}: \text{Kraichnan} \\ \frac{5}{3}: \text{Kolmogorov} \\ 2: \text{hard sphere} \end{cases}$$

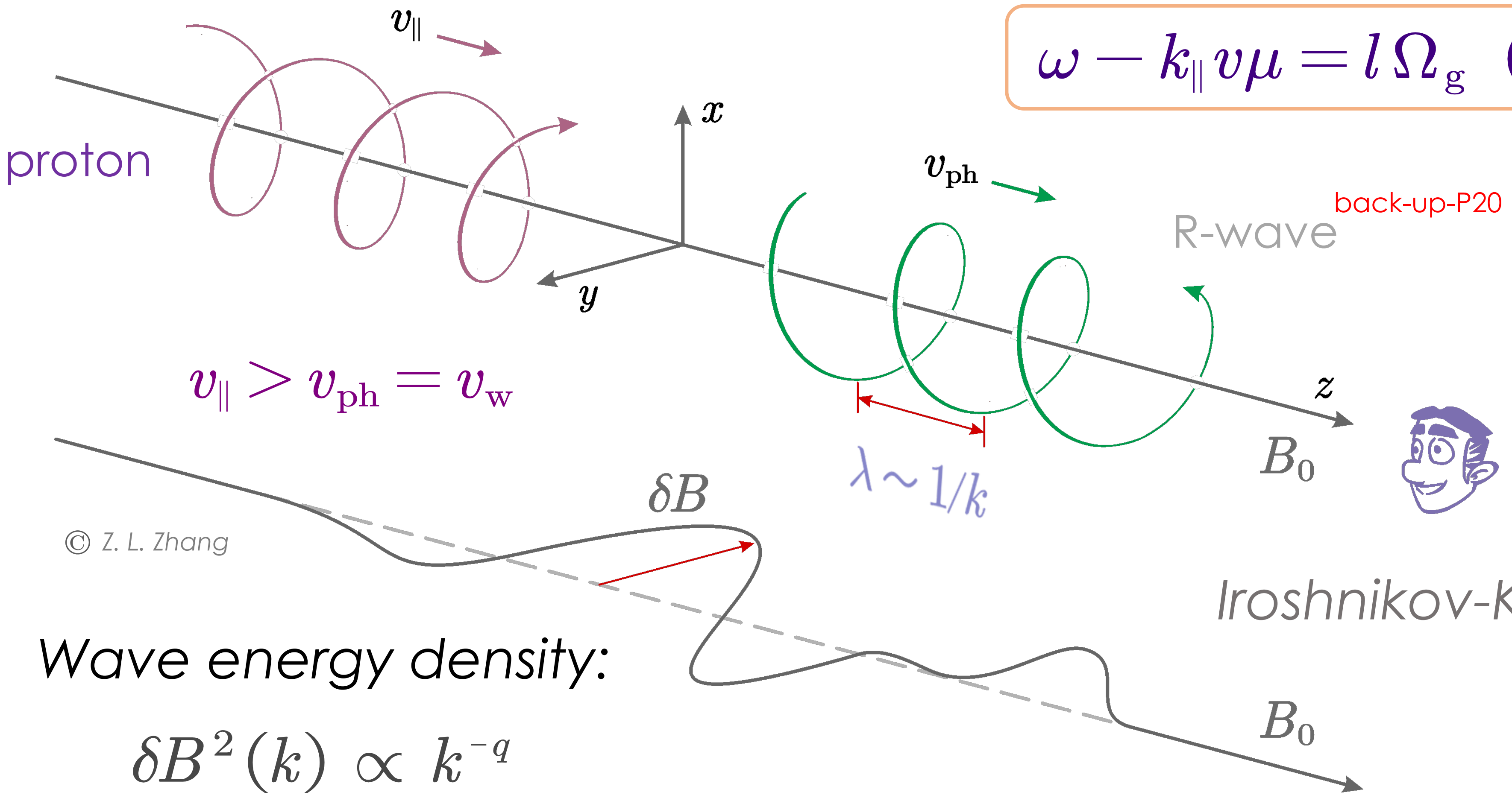
Interactions between turbulence and particles

Anomalous resonance: right (left)-handed waves interact with left (right)-handed protons

$$\omega - k_{\parallel} v \mu = l \Omega_g \quad (l = 0, \pm 1, \pm 2 \ldots)$$

$l = -1$

Important !



Cho & Lazarian 2003

Makwana & Yan 2020

Wave energy density:

$$\delta B^2(k) \propto k^{-q}$$

Iroshnikov-Kraichnan-like (IK-like):

$$W(k) \propto k^{-3/2}$$

Fast mode waves

The kinetic energy and magnetic energy in fluctuation are comparable.

Stochastic acceleration by turbulence

Zhou & *Matthaeus* 1990
Jun Kakuwa 2016
Petrosian & Liu 2004
Stawarz & Petrosian 2008

Fokker-Planck equation (fast mode waves):

back-up--P21,23,24,26

Cooling of waves — Injection of waves

$$\frac{\partial W}{\partial t} = \frac{\partial}{\partial k}\left[\mathcal{D}_{\rm kk}(k,t)\frac{\partial W}{\partial k}\right] - \frac{\partial}{\partial k}\left[\frac{2\mathcal{D}_{\rm kk}(k,t)}{k}W\right] + \frac{k}{3}(\nabla\cdot\mathbf{v})\frac{\partial W}{\partial k} + \Gamma_{\rm w}(k,t)W + \mathcal{Q}_{\rm w,inj}(k,t)$$

Diffusion and Convection: Cascading of waves — Damping of waves

Fokker-Planck equation of protons:

back-up--P25

Cooling — Source injection

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial E}\left[\mathcal{D}_{\rm EE}(E,t)\frac{\partial N}{\partial E}\right] - \frac{\partial}{\partial E}\left[\left(\frac{2\mathcal{D}_{\rm EE}(E,t)}{E} + \langle\dot{E}\rangle\right)N\right] - \frac{N}{t_{\rm esc}} + \mathcal{Q}_{\rm inj}(E,t)$$

Diffusion — Acceleration (Convection) — Escaping

Energy dissipation rate of turbulent waves = Energy gain rate of protons back-up-P24

Evolution of GRB ejecta in the early afterglows

The expansion of GRB remnants

$$\frac{d\Gamma_s}{dm} = -\frac{\Gamma_s^2 - 1}{M_{ej} + \epsilon m + 2(1-\epsilon)\Gamma_s m}$$

$$\simeq -\frac{\Gamma_s^2 - 1}{M_{ej} + 2\Gamma_s m}$$ (adiabatic case $\epsilon = 0$)

M_{ej}: mass ejected from GRB central engine

m: rest mass of the swept-up medium

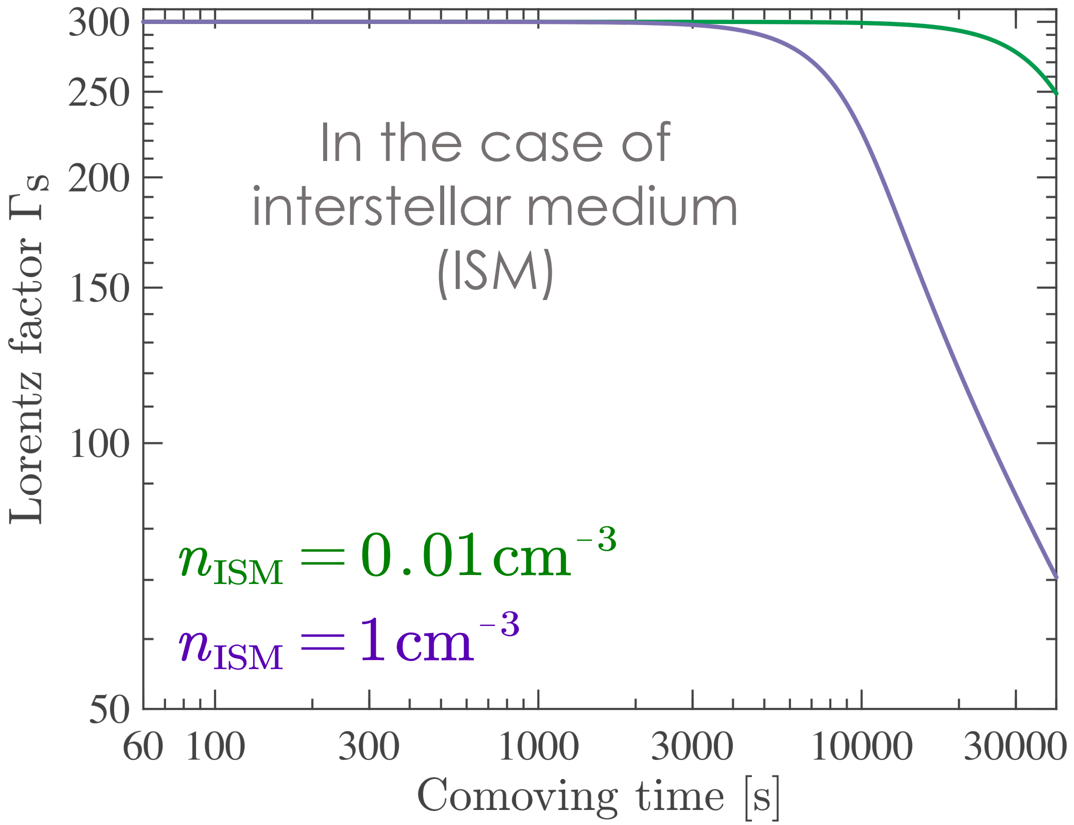
$$dm = 4\pi R^2 n_{ISM} m_p dR$$

R: radius of the blast wave

$$dR = \beta_{sh} c\Gamma_s \left(\Gamma_s + \sqrt{\Gamma_s^2 - 1}\right) dt$$

β_w: bulk velocity of the material

Y. F. Huang *et al.* MNRAS 1999



Timescales

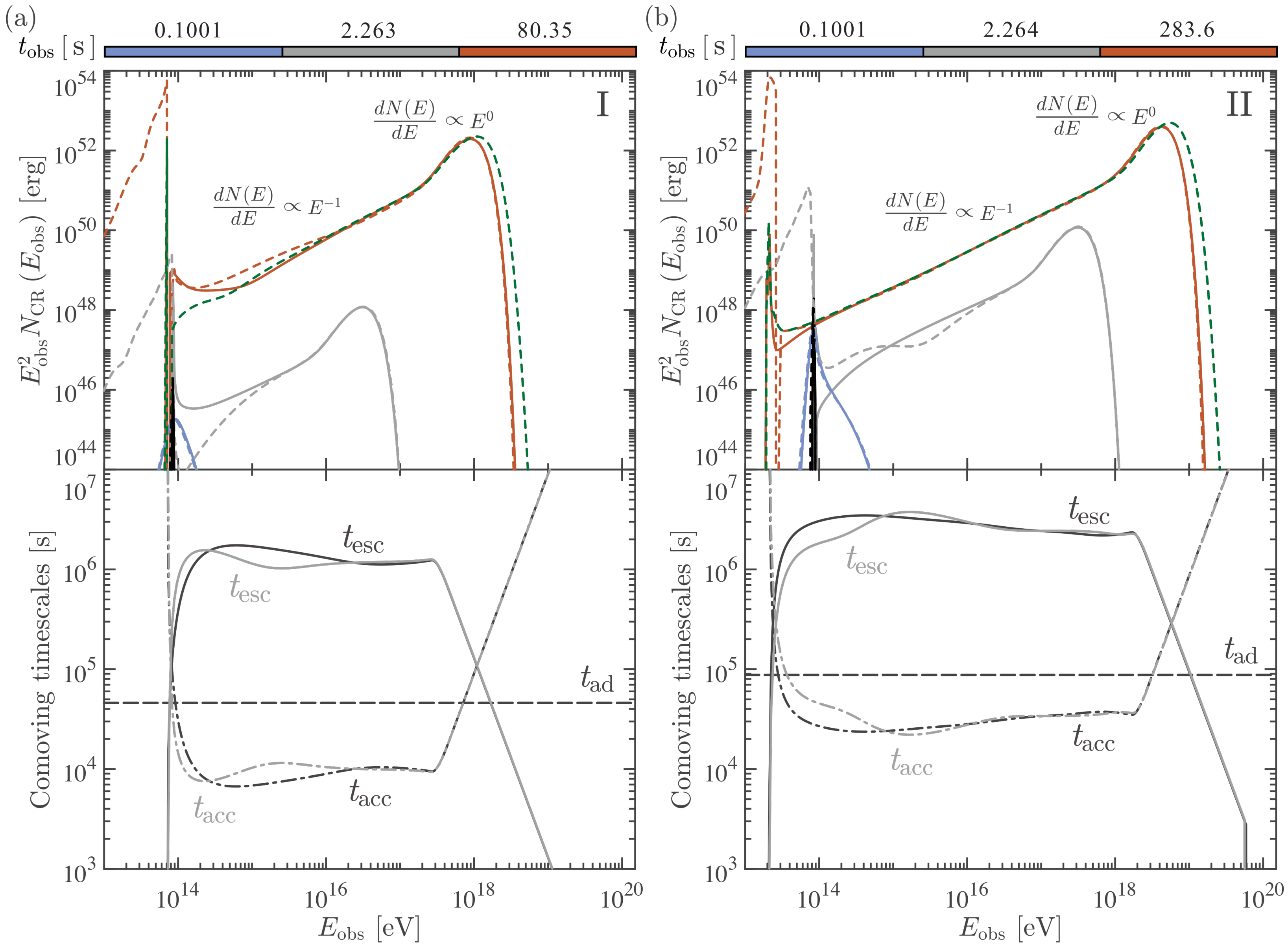
$$t_{esc} \approx R^2/\mathcal{D}_{RR} \approx R^2/(\Gamma^2 v_w^2 t_{acc})$$

$$\mathcal{D}_{RR}\mathcal{D}_{EE} \approx \beta_w^2 E^2 \qquad t_{acc} \approx E^2/\mathcal{D}_{EE} \qquad t_{ad} = R/\Gamma c$$

Results and discussions

Particle spectra

$t_{\rm cmv} = 40000\,{\rm s}$



case I $\quad \lambda_{\rm inj} = \frac{1}{k_{\rm inj}} = \frac{\xi R}{\Gamma} \lesssim \frac{R}{\Gamma}$

$\xi = 0.1,\ n_{\rm ISM} = 0.01\ {\rm cm}^{-3}$

case II

$\xi = 0.1,\ n_{\rm ISM} = 1\ {\rm cm}^{-3}$

Adiabatic cooling softens the spectrum at the cutoff regime

$t_{\rm acc} \simeq t_{\rm ad}$ *or* $E \simeq E_{\rm eq}$

Diffusive escape only play an important role in shaping the spectrum around

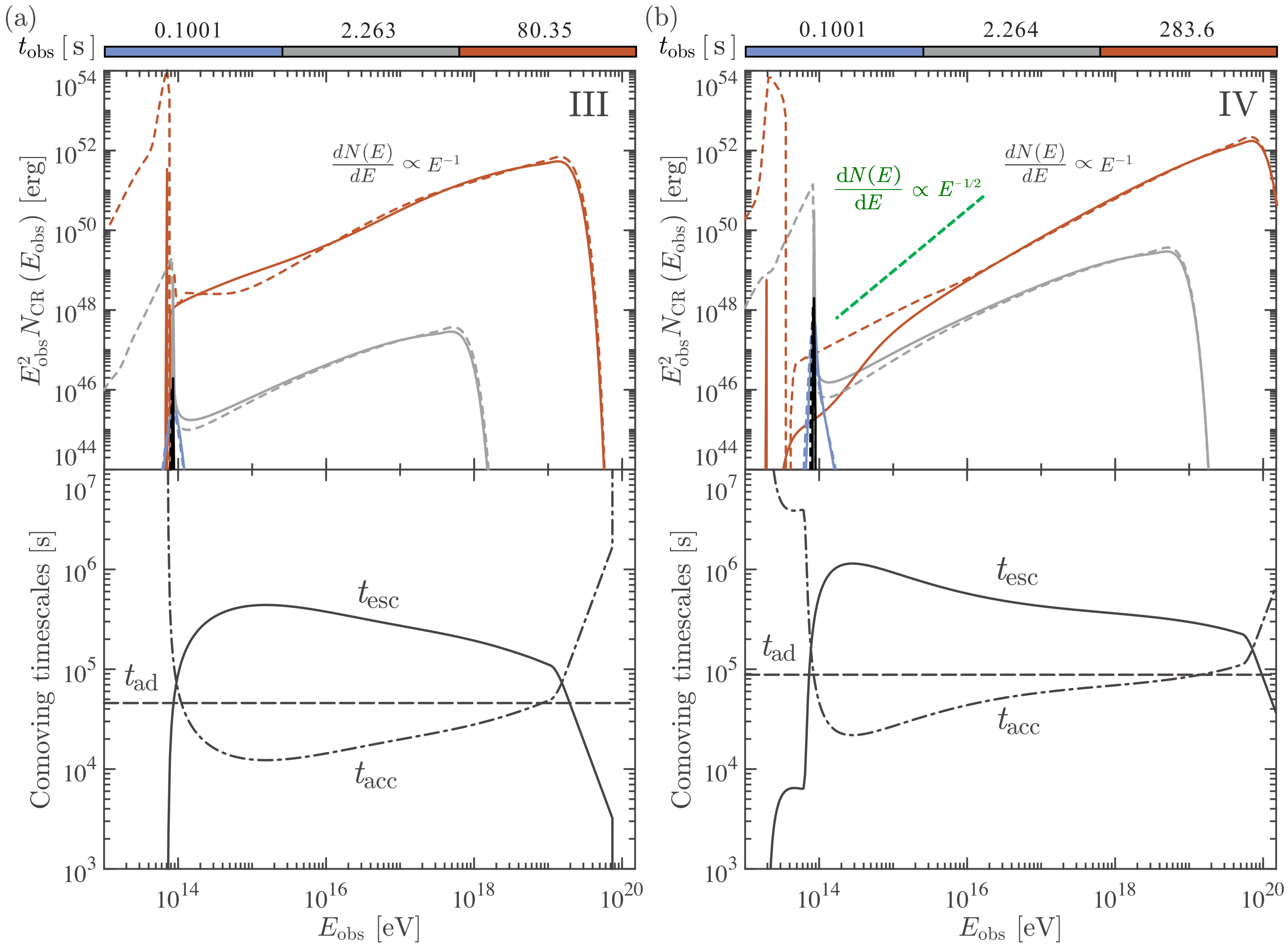
$E \simeq E_{\rm inj}$

Unearned baryon loading factor

Results and discussions

$t_{\rm cmv} = 40000\,{\rm s}$

Particle spectra



case III $\quad \lambda_{\rm inj} = \frac{1}{k_{\rm inj}} = \frac{\xi R}{\Gamma} \lesssim \frac{R}{\Gamma}$

$\xi = 1,\ n_{\rm ISM} = 0.01\ {\rm cm}^{-3}$

case IV

$\xi = 1,\ n_{\rm ISM} = 1\ {\rm cm}^{-3}$

If $q = \frac{3}{2}$, the steady state

$$\frac{{\rm d}N}{{\rm d}E} \propto E^{1-q},\ E \in (E_{\rm inj}, E_{\rm eq})$$

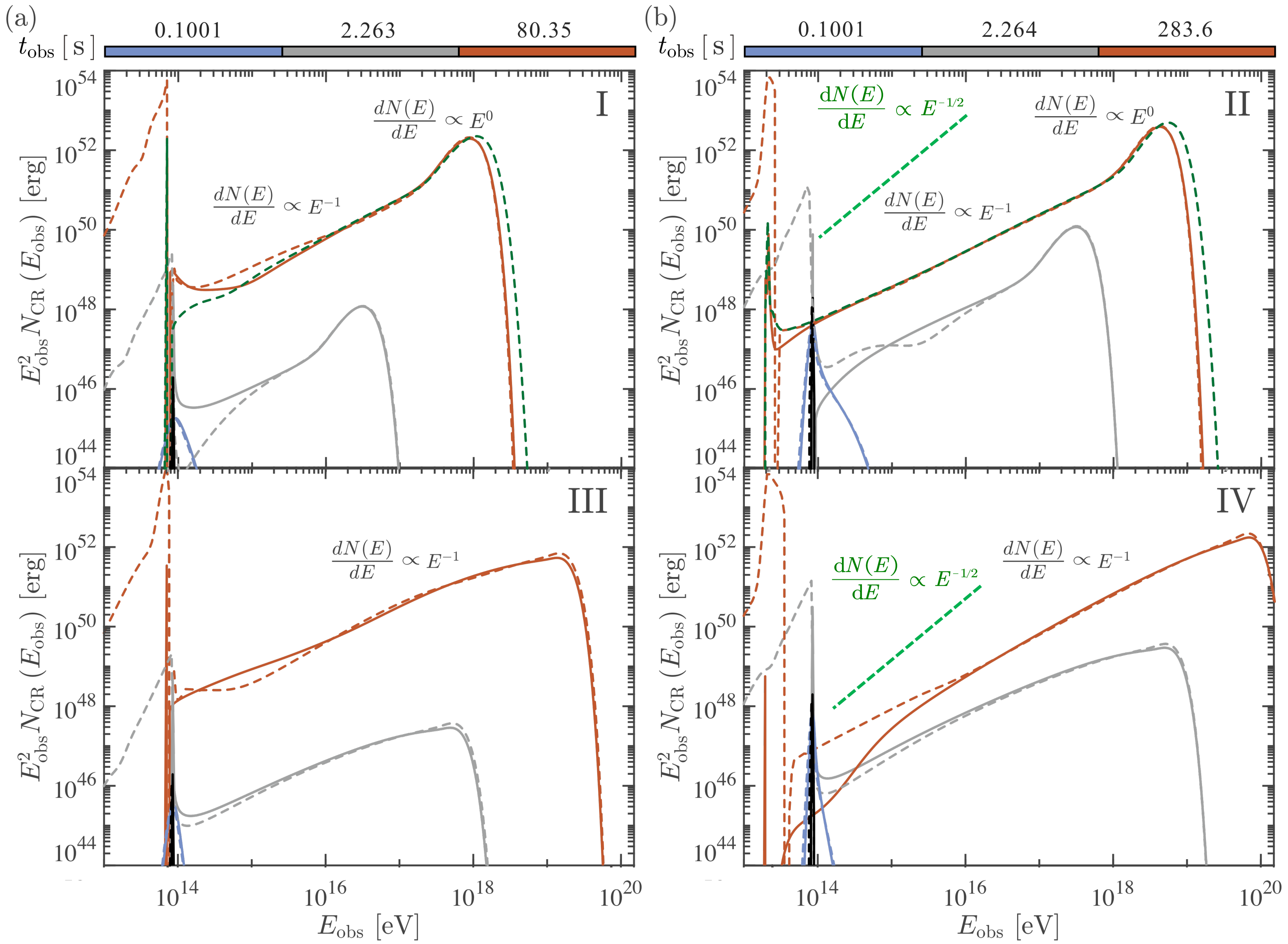
$$E^2 \frac{{\rm d}N}{{\rm d}E} \propto E^{3/2},\ (t_{esc} \gg t_{acc}, t_{ad})$$

Stawarz & Petrosian 2008

Results and discussions

$t_{\mathrm{cmv}} = 40000\,\mathrm{s}$

Particle spectra

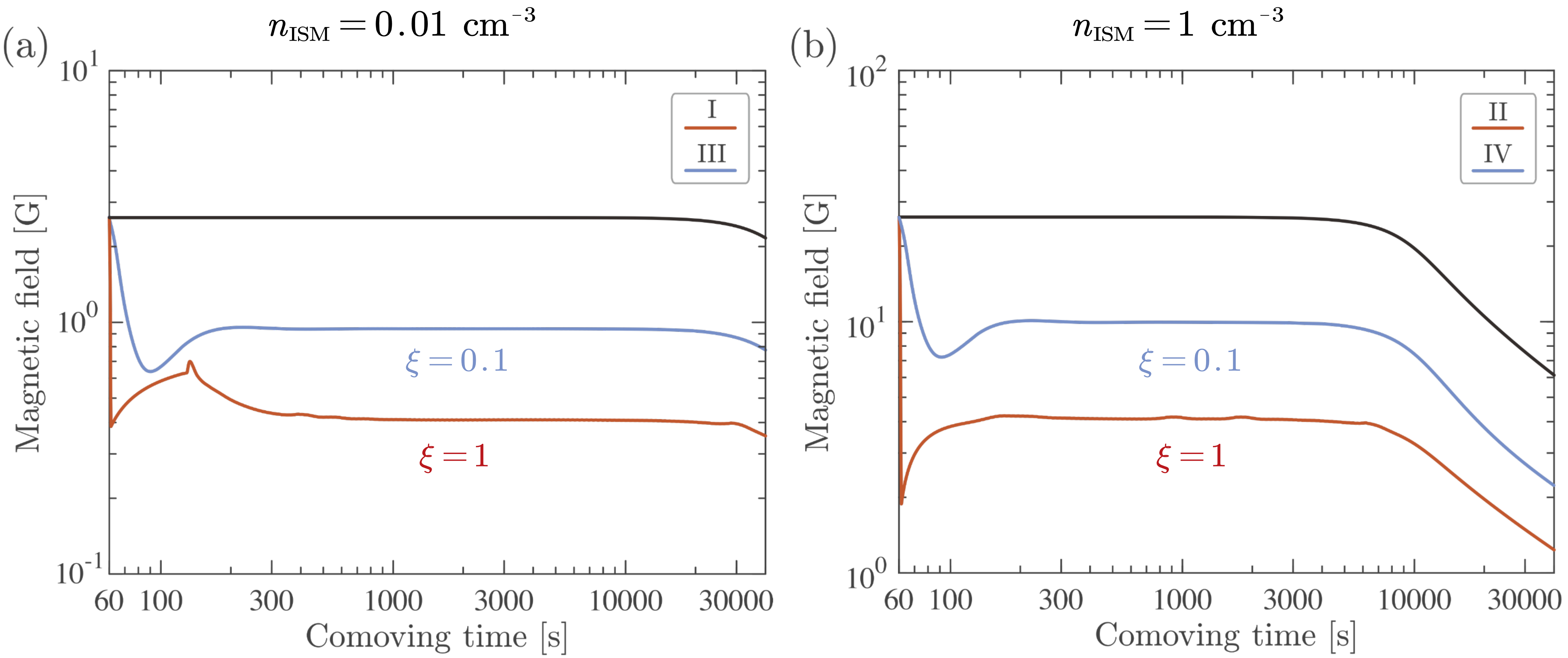


1. A smaller ξ leads to a hardening or a pile-up spectral feature at the high-energy end

2. A higher ISM density can converts more kinetic energy into the magnetic fields

Results and discussions

Magnetic field



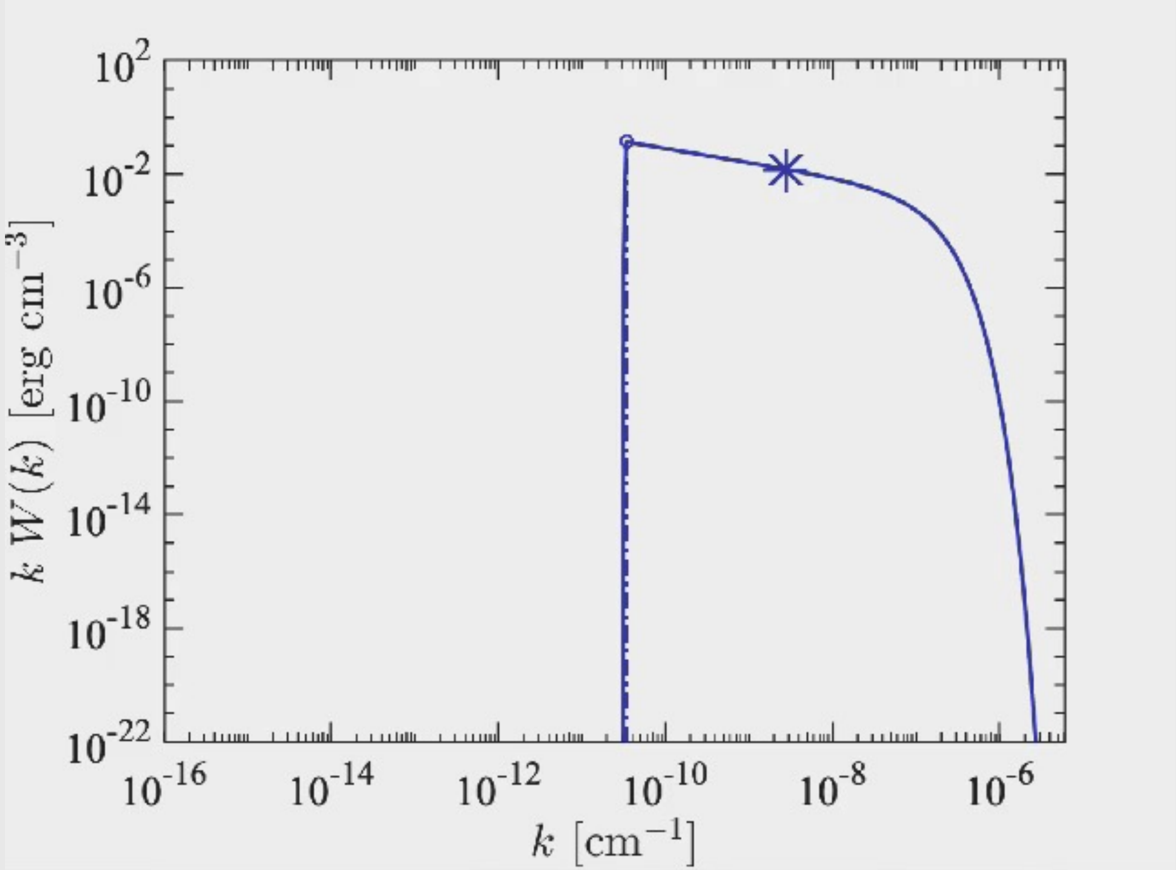
$$\delta B \simeq \sqrt{8\pi k W_{\rm B}},\ B = \sqrt{32\pi\varepsilon_B \Gamma^2 n_{\rm ISM} m_{\rm p} c^2}$$

Results and discussions

Magnetic field

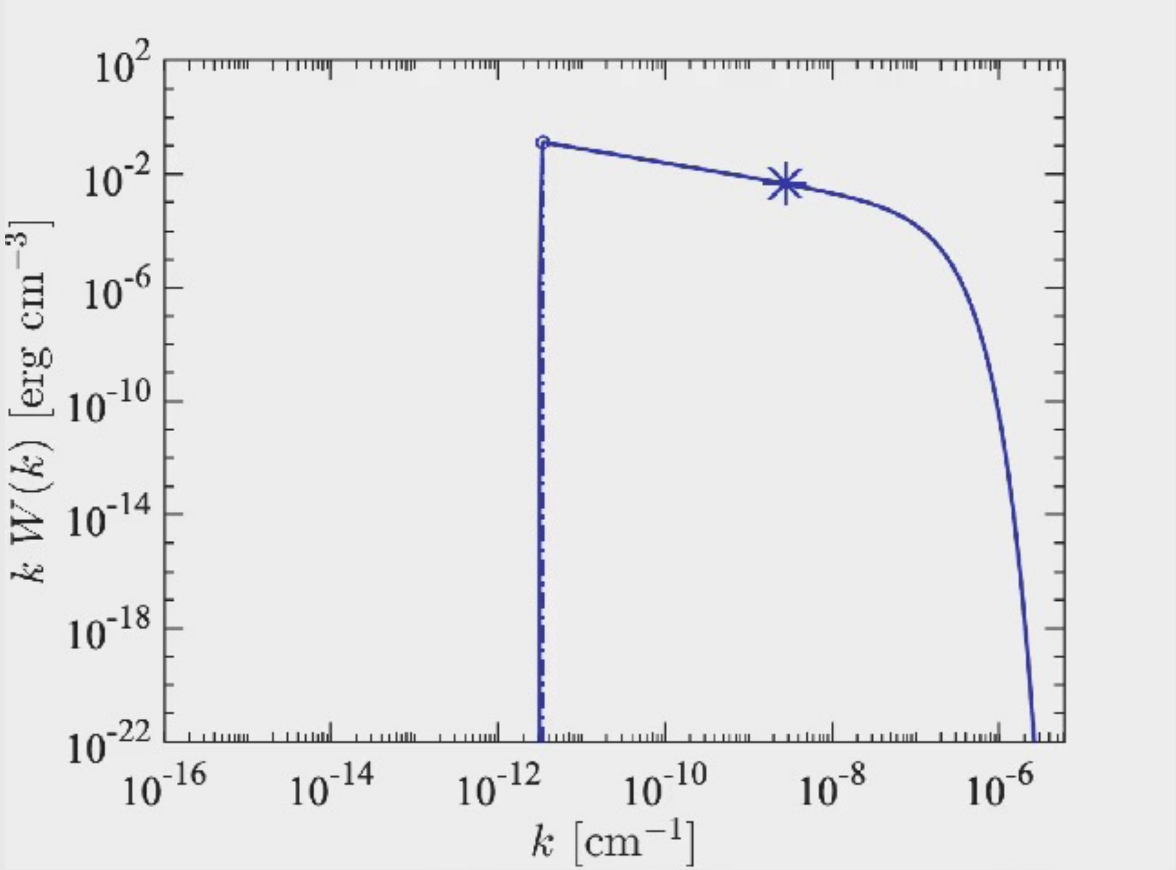
$$t_{\rm cmv} = 4500\,{\rm s}$$

$t_{\rm obs} \in [0.1001\,{\rm s},\ 0.2697\,{\rm s}]$



case I

$t_{\rm obs} \in [0.1001\,{\rm s},\ 0.2550\,{\rm s}]$



case III

Results and discussions

Photodisintegration of UHECRs in GRBs

Spectrum of X-rays in early afterglows: $F_\nu \sim \nu^{-1}$

Photodisintegration rate

fast cooling

$$t_{\rm dis}^{-1} = \frac{4}{3} c\sigma_0 \frac{\Delta_{\rm GDR}}{\varepsilon_0'} \frac{\Gamma_{\rm A} U_{\rm X}}{\kappa \varepsilon_0'}$$

X. Y. Wang *et al.* 2008

$\sim 68 - 100{\rm s}$

$$U_{\rm X} = \kappa n_{\rm b}(\varepsilon_{\rm b})\varepsilon_{\rm b}^2$$

i.e. GRB 190114C

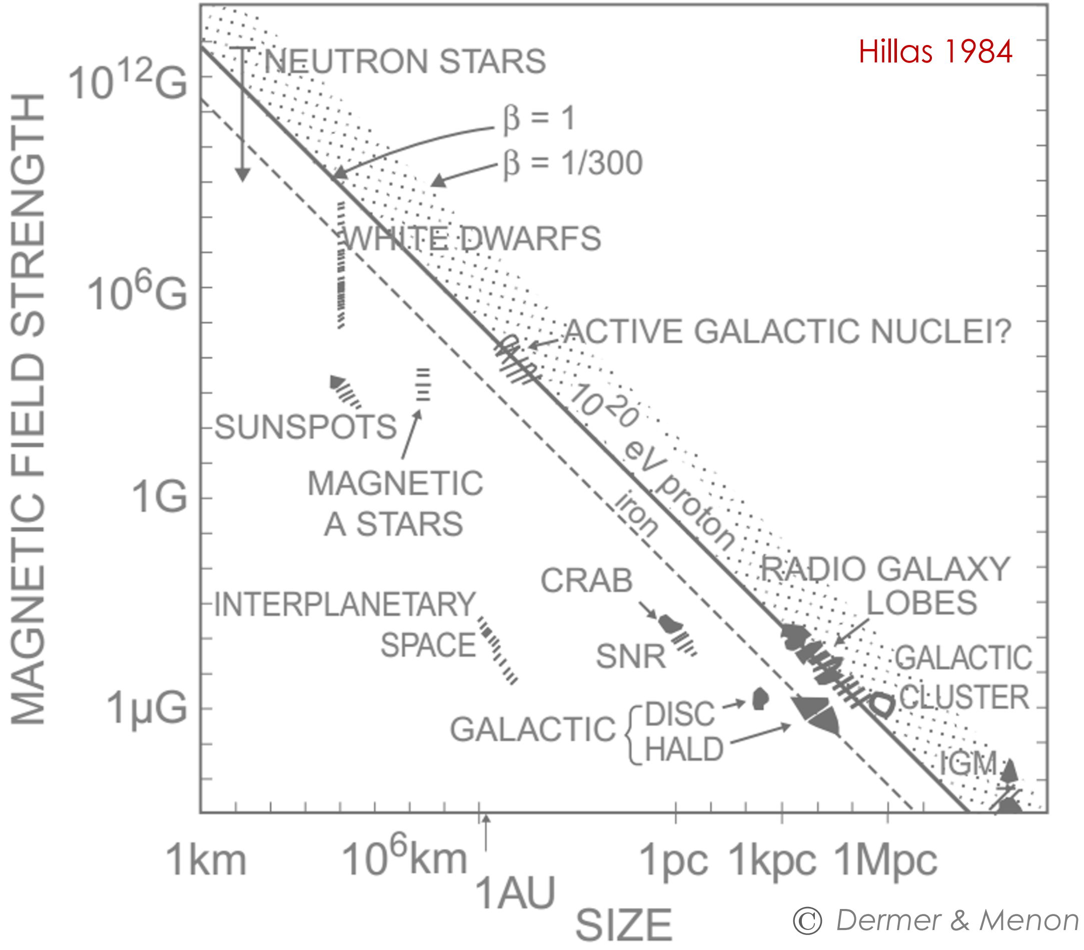
$$L_{\rm X} = 4\pi R_{\rm ex}^2 \Gamma^2 c U_{\rm X} \simeq 10^{48.5} {\rm erg\ s^{-1}}$$

Effective optical depth

$t_{\rm obs} \simeq 80{\rm s}$

$\tau = t_{\rm dyn}/t_{\rm dis}$ $\ll 1$

$$= \begin{cases} 6.5\times10^{-5} L_{\rm X,48.5} E_{\rm obs,18} R_{\rm ex,17.5}^{-1} \Gamma_{249}^{-4} (A/56)^{0.42} & \rm I \\ 6.5\times10^{-4} L_{\rm X,48.5} E_{\rm obs,19} R_{\rm ex,17.5}^{-1} \Gamma_{249}^{-4} (A/56)^{0.42} & \rm III \\ 9.1\times10^{-3} L_{\rm X,48.5} E_{\rm obs,18.5} R_{\rm ex,17.1}^{-1} \Gamma_{123}^{-4} (A/56)^{0.42} & \rm II \\ 9.1\times10^{-2} L_{\rm X,48.5} E_{\rm obs,19.5} R_{\rm ex,17.1}^{-1} \Gamma_{123}^{-4} (A/56)^{0.42} & \rm IV \end{cases}$$



Hillas criterion

$\varepsilon_{\rm p} \sim Ze\beta BR$

back-up-27

$\varepsilon_{\rm p} \sim 10^{19}{\rm eV} \Rightarrow \varepsilon_{\rm CRs} \sim 10^{20}{\rm eV}$

Summary

- ⋆ Including the evolution of jet's dynamics can weaken the capacity of the acceleration of the SA mechanism;
- ⋆ It also results in a particle spectrum softer than that predicted in the test-particle limit;
- ⋆ Protons can nevertheless be accelerated up to $10^{19}\mathrm{eV}$ with a spectrum $\mathrm{d}N/\mathrm{d}E \propto E^{-1}$ for some choices of system's parameters;
- ⋆ UHE nuclei can survive photodisintegration in early afterglows of GRBs then intermediate-mass nuclei can achieve $10^{20}\mathrm{eV}$ in GRB jets.

Thank you!
Comments are welcome!

Back-up

Interactions between turbulence and particles

Two types of resonant interactions:

①. $l = 0$: a. Landau (Cherenkov) damping — interact with oscillating $E_{\parallel}$

b. Transient time damping (TTD) — moving magnetic mirrors

②. $l \neq 0$: gyroresonance — interact with $E_{\perp}$

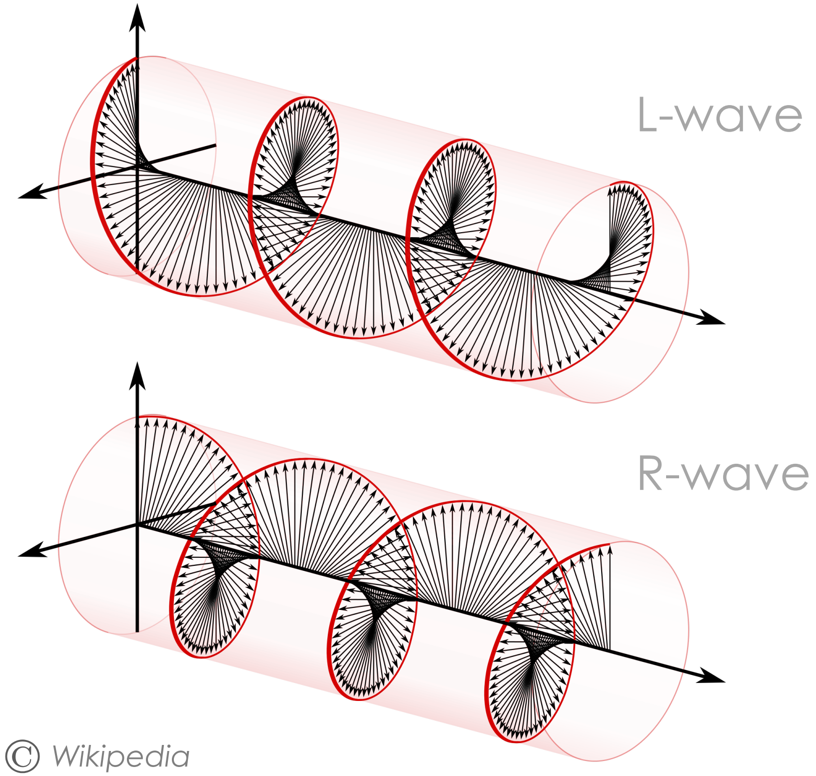
Resonant condition

$$\omega - k_{\parallel} v \mu = l\,\Omega_{\rm g} \quad (l = 0, \pm 1, \pm 2 \ldots)$$

$\Omega_{\rm g} = \Omega_0/\gamma$: gyrofrequency of relativistic particle

$\mu = v_{\parallel}/v$: pitch angle cosine

ω: wave frequency



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The diffusion equation for the waves

Miller *et al.* ApJ 1995, 1996

1. For isotropic *fast mode waves*

$$\frac{\partial W_{\mathrm{TFM}}}{\partial t} = \frac{\partial}{\partial k}\left[k^2 \mathcal{D}_{\mathrm{FM}} \frac{\partial}{\partial k}(k^{-2} W_{\mathrm{TFM}})\right] - \gamma_{\mathrm{FM}} W_{\mathrm{TFM}} + \mathcal{Q}_{\mathrm{FM}}$$

W_{TFM}: total (field + plasma motion) wave energy density per unit wavenumber *k*

$\mathcal{D}_{\mathrm{FM}}$: diffusion coefficient which describes the turbulent cascading

γ_{FM}: damping rate, due to electrons and protons

2. For *Alfvén waves (propagating only parallel and antiparallel to B_0)*

$$\frac{\partial W_{\mathrm{TA}}}{\partial t} = \frac{\partial}{\partial k_{\parallel}}\left(\mathcal{D}_{\parallel\parallel} \frac{\partial W_{\mathrm{TA}}}{\partial k_{\parallel}}\right) - \gamma_{\mathrm{A}} W_{\mathrm{TA}} + \mathcal{Q}_{\mathrm{A}}$$

W_{TA}: total Alfvén wave energy density per unit wavenumber *k*

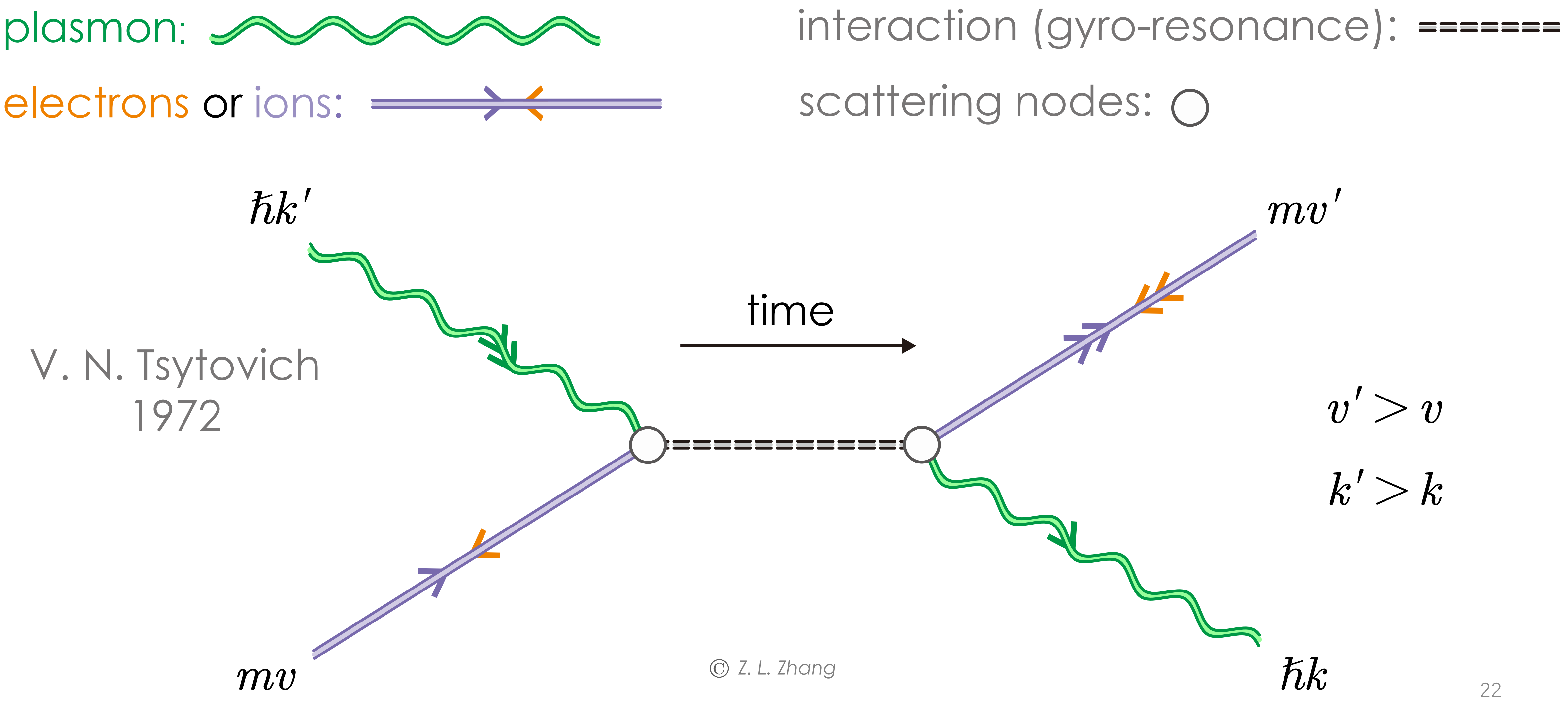
$\mathcal{D}_{\parallel\parallel}$: diffusion coefficient describing the cascade of energy of waves in $k_{\parallel}$- space

γ_{A}: damping rate, due to only the protons

Statistical description: plasmon

Wave Damping / Particle Acceleration

Feynman-like diagram of resonant wave-particle interaction



MHD turbulent cascading

Zhou & Matthaeus JGR 1990
Jun Kakuwa ApJ 2016

Fokker-Planck equation (fast mode waves):

③ $$\frac{\partial W}{\partial t} = \frac{\partial}{\partial k}\left[\mathcal{D}_{\mathrm{kk}}(k,t)\frac{\partial W}{\partial k}\right] - \frac{\partial}{\partial k}\left[\frac{2\mathcal{D}_{\mathrm{kk}}(k,t)}{k}W\right] + \frac{k}{3}(\nabla\cdot\mathbf{v})\frac{\partial W}{\partial k} + \Gamma_{\mathrm{w}}(k,t)W + \mathcal{Q}_{\mathrm{w,inj}}(k,t)$$

③† $\Gamma_{\mathrm{W}} = -\gamma_{\mathrm{FM}}$: damping rate, due to protons

$\mathcal{Q}_{\mathrm{w,inj}}$: continuous injection of waves at a mono-scale $\lambda_{\mathrm{inj}} = 1/k_{\mathrm{inj}}$

$$W_{\mathrm{B}}(k) = \frac{W(k)}{2}$$ ④

③ $\mathcal{D}_{\mathrm{kk}} = C^2 k^4 v_{\mathrm{w}}\left[\frac{W(k)}{2u_{\mathrm{B}}}\right]$: diffusion coefficient

②† C^2: Kolmogorov constant ~1

The comoving mean magnetic field:

⑤ $$u_{\mathrm{B}} = \frac{B^2}{8\pi} = \int_{k_{\mathrm{min}}}^{k_{\mathrm{max}}} W_{\mathrm{B}}(k)\,\mathrm{d}k$$

The wave spectrum (initial condition):

$$W(k,t=0) \equiv \kappa_0 u_{\mathrm{B}}\left(\frac{k}{k_{\mathrm{inj}}}\right)^{-q}\exp\left(-\frac{k}{k_{\mathrm{max}}}\right)$$ ②

$$\kappa_0 \approx -2k_{\mathrm{inj}}^{q}\left(k_{\mathrm{max}}^{-q+1} - k_{\mathrm{inj}}^{-q+1}\right),\ q = \frac{3}{2}$$

① Initial ambient magnetic field:

$$B_0 \simeq \sqrt{32\pi\varepsilon_B\Gamma_0^2 n_{\mathrm{ISM}} m_{\mathrm{p}} c^2}$$

MHD waves damping

Brunetti & Lazarian 2007

Jun Kakuwa 2016

Fokker-Planck equation (fast mode waves):

$$\frac{\partial W}{\partial t} = \frac{\partial}{\partial k}\left[\mathcal{D}_{\rm kk}(k,t)\frac{\partial W}{\partial k}\right] - \frac{\partial}{\partial k}\left[\frac{2\mathcal{D}_{\rm kk}(k,t)}{k}W\right] + \frac{k}{3}(\nabla\cdot\mathbf{v})\frac{\partial W}{\partial k} + \Gamma_{\rm w}(k,t)W + \mathcal{Q}_{\rm w,inj}(k,t)$$

Energy dissipation rate of turbulent waves = Energy gain rate of protons

$$\int {\rm d}k\,\Gamma_{\rm w}(k)W(k) = -\int {\rm d}E\,E\frac{\partial F_{\rm p}(E)}{\partial E},$$

$$F_{\rm p}(E) = E^2\mathcal{D}_{\rm EE}(E)\frac{\partial}{\partial E}\left[\frac{N(E)}{E^2}\right]$$

Integrating by parts twice, we get

$$\Gamma_{\rm w}(k) = -\frac{4\pi e^2\beta_{\rm w}^2 c}{k}\left[N(E_{\rm res}(k)) + \int_{E_{\rm res}(k)}^{E_{\rm max}}\frac{2N(E)}{E}{\rm d}E\right], \qquad E_{\rm res}(k) = \frac{eB}{k}$$

The turbulence at *k* is damped by protons with energy $E > E_{\rm res}(k)$.

Diffusion approximation

Diffusion coefficient in energy space:

In numerical calculation, assuming $v \gg v_\mathrm{w}$

$$\mathcal{D}_\mathrm{EE}(E) \approx \pi E^2 \left(\frac{v_\mathrm{w}}{v}\right)^2 \frac{|\Omega_\mathrm{g}|}{r_\mathrm{g} B^2} \int_{1/r_\mathrm{g}}^{k_\mathrm{max}} \frac{\mathrm{d}k_\parallel}{k_\parallel} \left[1 - \frac{1}{(r_\mathrm{g} k_\parallel)^2}\right] B^2(k_\parallel)$$

$$\sim \frac{E^2 \beta_\mathrm{w}^2 k_\mathrm{res} c}{r_\mathrm{g} u_\mathrm{B}} \int_{k_\mathrm{min}}^{k_\mathrm{max}} k^{-1} W_\mathrm{B}(k)\mathrm{d}k \qquad \sim 1$$

$$\sim \frac{E^2 \beta_\mathrm{w}^2 c}{r_\mathrm{g}(E)} \zeta(k_\mathrm{res}) \qquad \zeta \equiv \frac{kW_\mathrm{B}(k)}{u_\mathrm{B}} \sim 1$$

The lower end of the integration is more important than the upper end, so wavenumber

$$k_\parallel \sim 1/r_\mathrm{g}(E) = k_\mathrm{res}$$

Mainly contributes to $\mathcal{D}_\mathrm{P}^\mathrm{L}$ or $\mathcal{D}_\mathrm{EE}$.

$$\mathcal{D}_\mathrm{ij} = \mathcal{D}_\mathrm{P}^\mathrm{L} \frac{p_\mathrm{i} p_\mathrm{j}}{p^2} + \mathcal{D}_\mathrm{P}^\mathrm{T} \left(\delta_\mathrm{ij} - \frac{p_\mathrm{i} p_\mathrm{j}}{p^2}\right)$$

$\mathcal{D}_\mathrm{P}^\mathrm{L}$: Longitudinal diffusion coefficient

$\mathcal{D}_\mathrm{P}^\mathrm{T}$: Transverse diffusion coefficient

anisotropy $\xrightarrow{\mathcal{D}_\mathrm{P}^\mathrm{T} \gg \mathcal{D}_\mathrm{P}^L}$ isotropy

$$\mu = \frac{p_\parallel}{p} = \frac{m\Omega/k_\parallel}{m\Omega/k} = \frac{k}{k_\parallel}$$

For large pitch-angle, $\mu \to 0$

The particles keep isotropy by pitch-angle scattering on a much shorter timescale than the momentum diffusion.

$$k_\mathrm{min} = \frac{2\pi}{\lambda_\mathrm{inj}},\ k_\mathrm{max} = \frac{2\pi}{\lambda_\mathrm{min}},\quad \lambda_\mathrm{inj} = \frac{1}{k_\mathrm{inj}} = \frac{\xi R}{\Gamma} \lesssim \frac{R}{\Gamma}$$

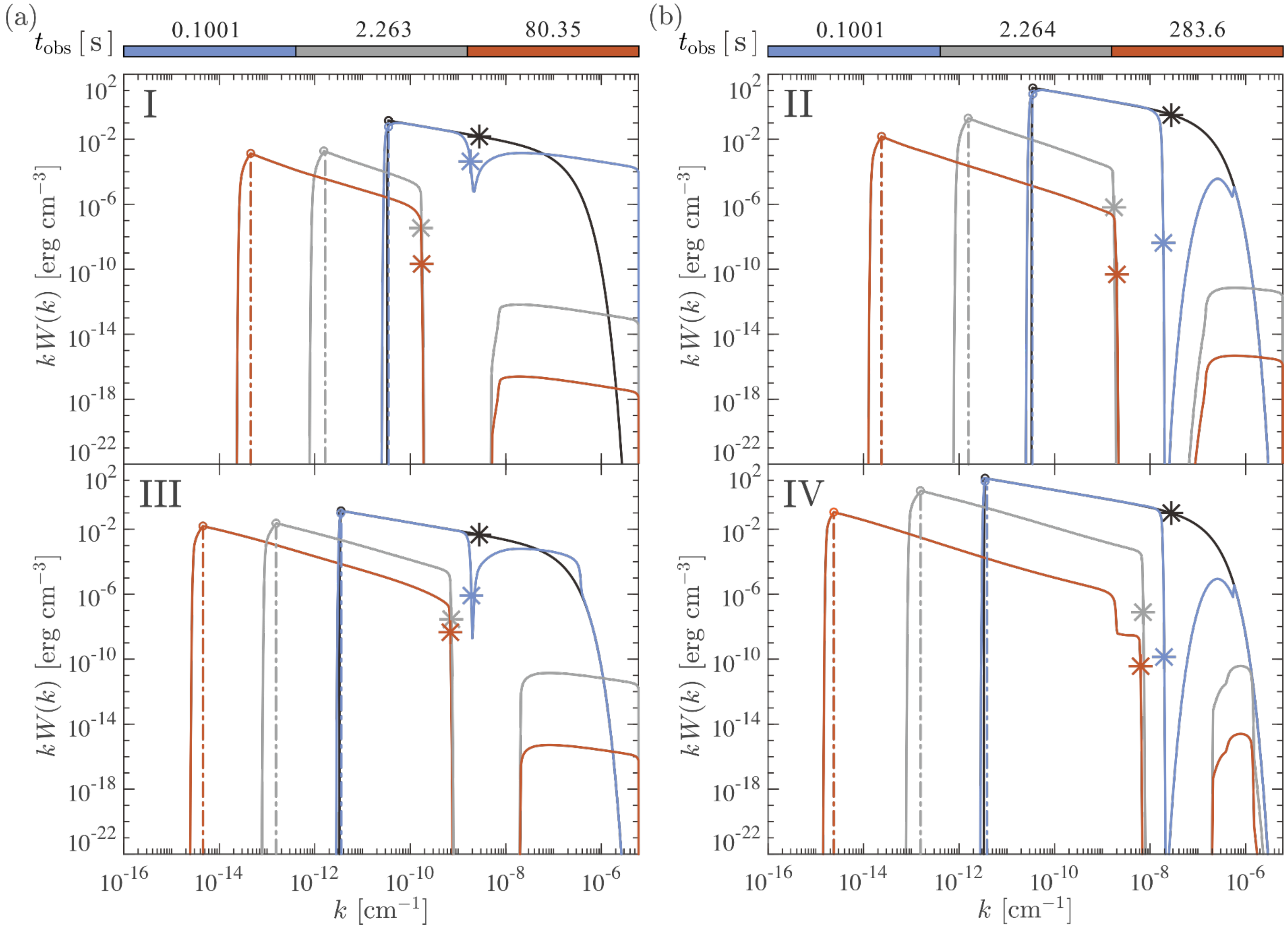
$0 < \xi \lesssim 1$: Dimensionless parameter

Turbulent wave spectra

$$t_{\rm cmv} = 40000\,{\rm s}$$

$$k_{\rm inj} = \frac{1}{\lambda_{\rm inj}}$$

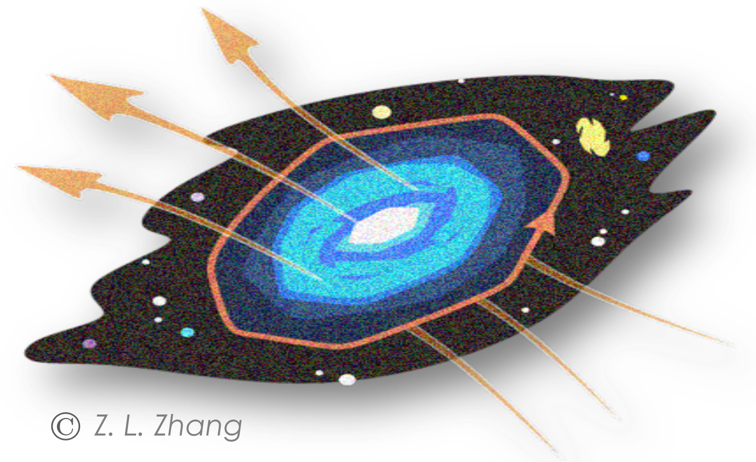
$$k_{\rm res,\,inj} = \frac{eB}{E_{\rm res}} = \frac{eB}{E_{\rm inj}}$$



The larger k associated eddies have already been damped by the corresponding lower E particles.

Then E transport in k-space will cause more remarkable deviation from IK-type spectrum in lower k.

Hillas criterion



Faraday's law of induction

$$\oint_l \vec{E} d\vec{l} = -\frac{1}{c}\frac{\partial}{\partial t}\iint_s \vec{B}\cdot d\vec{S}$$

$$U = \frac{1}{c}\frac{d\Phi}{dt} \sim \frac{1}{c}\frac{BR^2}{R/v} = \beta BR$$

potential difference

$$\varepsilon_{\mathrm{p}} \sim Ze\beta BR$$