Parameterization of Muon Production Profiles in the Atmosphere

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Introduction

Elbert formula: estimate high-energy muon multiplicity in air shower

$$\langle N_\mu(> E_\mu, E_0, A, \theta)\rangle \approx A \times \frac{0.0145\,\text{TeV}}{E_\mu \; \cos\theta} \left(\frac{E_0}{A\,E_\mu}\right)^{0.757} \left(1 - \frac{A\,E_\mu}{E_0}\right)^{5.25}$$

Limitations:

- No production depth / altitude
- No atmospheric conditions

Goal: parameterize longitudinal muon production profiles from simulation

Muon production fit formula

Muon production differential in slanth depth

$$\left\langle \frac{\mathrm{d}N}{\mathrm{d}X}(X, T, E_0, A, E_\mu, \theta) \right\rangle = \text{Derivative of Gaisser-Hillas function} \quad (1)$$

$$\times \quad \text{Relative probability of meson decay to muons} \quad (2)$$

$$\times \quad \text{Threshold factor from Elbert formula} \quad (3)$$

X	slanth depth
T	atmospheric temperature
E_0	primary energy
A	primary mass
E_μ	minimum muon energy
θ	zenith angle

Muon production fit formula (1)

$$\left\langle \frac{\mathrm{d}N}{\mathrm{d}X}(X, T, E_0, A, E_\mu, \theta) \right\rangle = N_{max} \times \exp\left((X_{max} - X)/\lambda\right) \times \left(\frac{X_0 - X}{X_0 - X_{max}}\right)^{(X_{max} - X_0)/\lambda} \times \frac{X_{max} - X}{\lambda(X - X_0)}$$

$$\times \quad \text{Relative probability of meson decay to muons}$$

$$\times \quad \text{Threshold factor from Elbert formula}$$

- Interpreted as the production of charged mesons
- Four free parameters: $N_{max}, X_{max}, \lambda, X_0$

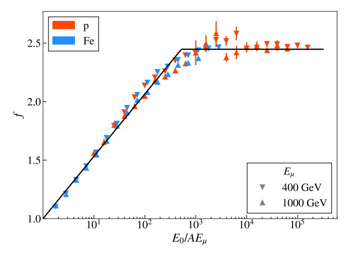
Muon production fit formula (2)

$$\left\langle \frac{\mathrm{d}N}{\mathrm{d}X}(X, T, E_0, A, E_\mu, \theta) \right\rangle = N_{max} \times \exp\left((X_{max} - X)/\lambda\right) \times \left(\frac{X_0 - X}{X_0 - X_{max}} \right)^{(X_{max} - X_0)/\lambda} \times \frac{X_{max} - X}{\lambda(X - X_0)}$$

$$\times \left[0.92 \times \frac{r_\pi \lambda_\pi \epsilon_\pi}{f E_\mu \cos(\theta) X} \times \frac{1}{1 + \frac{r_\pi \lambda_\pi \epsilon_\pi}{f E_\mu \cos(\theta) X}} + 0.08 \times \frac{r_K \lambda_K \epsilon_K}{f E_\mu \cos(\theta) X} \times \frac{1}{1 + \frac{r_K \lambda_K \epsilon_K}{f E_\mu \cos(\theta) X}} \right]$$

$\times$ Threshold factor from Elbert formula

- Fraction of decay vs decay & reinteraction $\frac{1/d_\pi}{1/d_\pi + 1/\lambda_\pi}$
- Decay length $\frac{1}{d_\pi} = \frac{\epsilon_\pi}{E_\pi \cos\theta X}$
- Critical energy $\epsilon_\pi = \frac{m_\pi c^2}{c\tau_\pi} \frac{RT}{Mg} \approx 115\,\text{GeV} \times \frac{T}{220\,\text{K}}$
- Average energy from decay: $E_\mu \approx r_\pi \times E_\pi$
- f: ratio between minimum energy E_μ and mean energy of muons above minimum
- Charged pions & kaons: 0.92 & 0.08 from momentum fractions and branching ratios



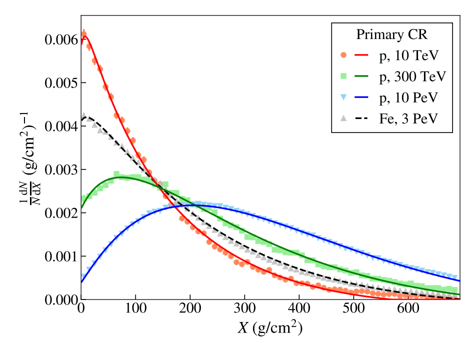
Muon production fit formula (3)

$$\left\langle \frac{\mathrm{d}N}{\mathrm{d}X}(X, T, E_0, A, E_\mu, \theta) \right\rangle = N_{max} \times \exp\left((X_{max} - X)/\lambda\right) \times \left(\frac{X_0 - X}{X_0 - X_{max}}\right)^{(X_{max} - X_0)/\lambda} \times \frac{X_{max} - X}{\lambda(X - X_0)}$$
$$\times \left[0.92 \times \frac{r_\pi \lambda_\pi \epsilon_\pi}{f E_\mu \cos(\theta) X} \times \frac{1}{1 + \frac{r_\pi \lambda_\pi \epsilon_\pi}{f E_\mu \cos(\theta) X}} + 0.08 \times \frac{r_K \lambda_K \epsilon_K}{f E_\mu \cos(\theta) X} \times \frac{1}{1 + \frac{r_K \lambda_K \epsilon_K}{f E_\mu \cos(\theta) X}}\right]$$
$$\times \left(1 - \frac{A E_\mu}{E_0}\right)^{5.99}$$

- Low energy behaviour (E_0/A close to E_μ)
- Exponent fit to our simulations

Fits to simulation

- CORSIKA simulations using Sibyll 2.3c
- Extract production of muons $> E_\mu$ per dX
- Fit with formula to obtain $N_{max}, X_{max}, \lambda, X_0$



Parameterization

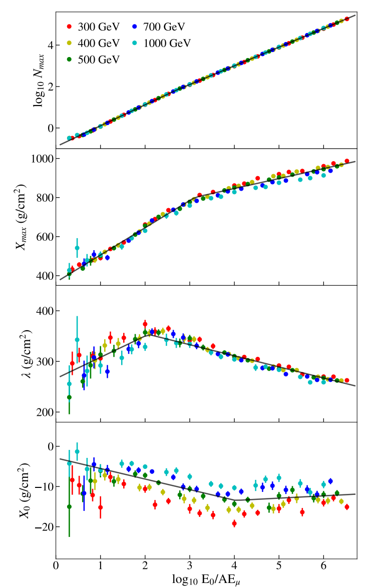
- Repeat for various primaries and thresholds E_μ
- Fit results depend in leading order on E_0/AE_μ
- Describe with*

$$N_{\mathrm{max}} = c_i \times A \times \left(\frac{E_0}{AE_\mu}\right)^{p_i}$$

$$X_{\mathrm{max}}, \lambda, X_0 = a_i + b_i \times \log_{10}\left(\frac{E_0}{AE_\mu}\right)$$

with $i = 1, 2$ below/above a break $E_0/AE_\mu = 10^q$

*actual parameters can be found in proceeding



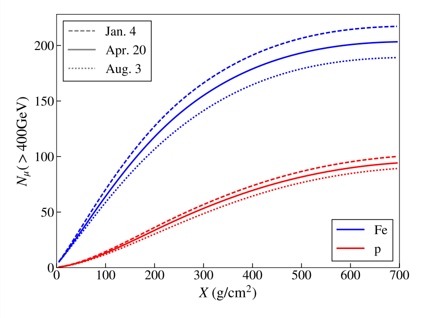
Application to muon bundles

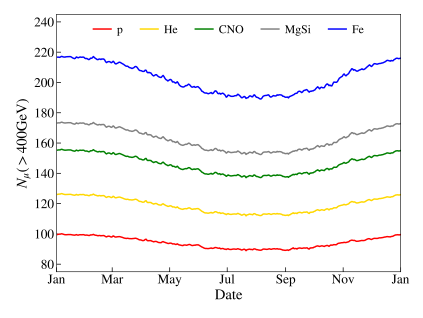
Example:

- Muon bundle at fixed primary energy
- Relevant for surface/underground coincidences
- Numbers used relevant for IceTop & IceCube:
  - elevation 2835 m a.s.l.
  - depth $\approx 700\,\mathrm{g/cm^2}$
  - vertical showers
  - $E_0 = 10\,\mathrm{PeV}$
  - $E_\mu = 400\,\mathrm{GeV}$
  - Atmospheric data from NASA AIRS satellite

Multiplicity variations

- Muon production profile obtained for realistic atmosphere
- Integrate for multiplicity
- Highest in (austral) summer, when atmosphere is warmest
- Seasonal variations $\sim 6\%$ around mean





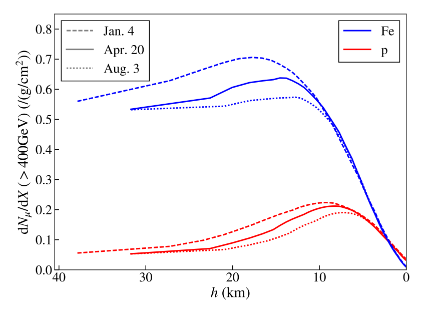
Production altitude

- Production altitude information from production depth

$$h(X_v) = \frac{RT}{Mg} \ln \frac{X_0}{X_v},$$

where X_0 is dept at $h = 0$

- Muons produced higher for heavier primaries
- Muons produced higher in summer

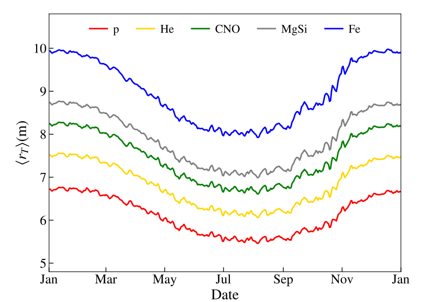


Size variations

- Production altitude combined with transverse momentum gives deviation at surface

$$r_T = \frac{p_T}{E_\mu} \times \frac{h}{\cos\theta},$$

- Estimate using $\langle p_T \rangle \approx 350\,\text{MeV}$
- $\langle r_T \rangle$: average of r_T weighted with production profile (geometric effect only)
- Largest muon bundle size in summer
- Seasonal variations $\sim 10\%$ around mean



Further applications

Estimation of rates of single/multiple muon events integrating over primary spectrum:

- PoS(ICRC2021)1202
- arXiv:2106.12247

The parameterization is also made available on GitHub:

- *https://github.com/verpoest/muon-profile-parameterization*

Note: Because the scaling with E_0/AE_μ is not perfect, it is best to optimize the parameters for a specific application

Summary

Parameterization of production profiles of high-energy muons in air showers:

- Muon production versus atmospheric depth
- Depends on atmospheric temperature
- Various applications:
    - muon multiplicity
    - muon bundle size
    - event rates
    - seasonal variations
    - ...

Thank you!